On the influence of constitutive relation in projectile impact of steel plates

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Abstract

In this paper the influence of constitutive relation has been studied in numerical simulations of the perforation of 12-mm thick Weldox 460 E steel plates impacted by blunt-nosed projectiles in the sub-ordinance velocity regime. A modified version of the well-known and much used constitutive relation proposed by Johnson-Cook and both the bcc- and hcp-version of the Zerilli-Armstrong constitutive relation were combined with the Johnson-Cook fracture criterion. These models were implemented as user-defined material models in the non-linear finite element code LS-DYNA. Identification procedures have been proposed, and the different models were calibrated and validated for the target material using available experimental data obtained from tensile tests where the effects of strain rate, temperature and stress triaxiality were taken into account. Perforation tests carried out in a compressed gas gun on 12-mm-thick circular Weldox 460 E steel plates were then used as base in a validation study of plate perforation using LS-DYNA and the proposed constitutive relations. The numerical study indicated that the physical mechanisms during perforation can be qualitatively well predicted by all constitutive relations, but quantitatively more severe differences appear. The reasons for this are discussed in some detail. It was concluded that for practical applications, the Johnson-Cook constitutive relation and fracture criterion seems to be a good choice for this particular problem and excellent agreement with the experimental results of projectile impact on steel plates were obtained under the conditions investigated.

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1. Introduction

In order to model the mechanical response of structures exposed to ballistic impact, it is necessary to establish reliable constitutive relations and fracture criteria as functions of large strains, high strain rates, temperature softening, varying stress states and history of loading in addition to accumulation of damage and mode of failure. A complete description that includes all these phenomena is however difficult to obtain, and
Nomenclature

- $a$ initial radius of the tensile specimen’s cross-section
- $a, p$ parameters in the Recht–Ipson model
- $A, B, C, n, m$ parameters in the Johnson–Cook constitutive relation
- $A, B, n, \sigma_a, x_0, x_1, \beta, \beta_0, \beta_1$ parameters in the Zerilli–Armstrong constitutive relation
- $C_p$ specific heat capacity
- CPU, $n$ CPU computational time, number of processors
- $d, d_0$ diameter of tensile specimen: current and initial
- $d_{plfr}$ plug diameter on the front side
- $d_{plre}$ plug diameter on the rear side
- $d_{tre}$ target’s cavity diameter on the rear side
- $D$ damage
- $D_i, D_f$ projectile diameter: initial and final, respectively
- $D_1, D_2, D_3, D_4, D_5$ parameters in the Johnson–Cook fracture model
- $\dot{e}_{avg}$ average nominal strain rate
- $E, E_t$ Young’s modulus, tangent modulus
- $El_{f, d}, El_{f, T^*}$ number of eroded elements due to: JC fracture criterion and temperature-based criterion
- $h_{pl}$ plug length
- $h_{pls}$ length of the plug’s shear zone
- $h_t$ target thickness
- $h_{tc}$ cavity length
- $h.m.s$ hours.minutes.seconds
- $L_i, L_f$ projectile length; initial and final, respectively
- $m_p$ projectile mass
- $m_{pl}$ plug mass
- $t_f$ time of complete perforation
- $R, R_0$ notch profile radius
- $T, T_0$ temperature, initial temperature
- $T_m, T_r, T^*$ melting, room and homologous temperature, respectively
- $T_{\text{max}}$ maximum temperature
- $v_{bl}$ ballistic limit velocity
- $v_i$ initial projectile velocity
- $v_r$ residual projectile velocity
- $v_{rpl}$ residual plug velocity
- $w_{\text{max}}$ maximum target deformation
- $w_{pl}$ plug deformation, i.e. $w_{pl} \approx h_{pl} \cdot h_{pls}$
- $\alpha$ thermal expansion coefficient
- $\Delta d$ diameter reduction of a tensile specimen
- $\Delta D$ projectile nose deformation, i.e. $\Delta D = D_f - D_i$
- $\Delta L$ projectile length reduction, i.e. $\Delta L = L_i - L_f$
- $e$ true strain
- $e_{eq}$ equivalent plastic strain
- $e_t$ true failure strain
- $e_u$ equivalent plastic strain at diffuse necking
- $\dot{e}_0$ user-defined reference strain rate
- $\dot{e}_{eq}$ equivalent plastic strain rate
- $\dot{e}_{eq}$ dimensionless strain rate
- $\nu$ Poisson’s ratio
- $\rho$ density
Certainly the model will become more complex by increasing the number of phenomena involved. To simplify the modelling it is often necessary to introduce assumptions according to the problem investigated.

Several constitutive relationships have been proposed for metallic materials under impact loading for use in computational mechanics that vary from being purely empirical to highly theoretical [1–3]. The empirical models are based on available experimental observations, while the latter are based on the microscopic nature of the material. The multiaxial stress state of the material is usually expressed in terms of the equivalent (von Mises) stress

\[ s_{\text{eq}} = f(e_{\text{eq}}, \dot{e}_{\text{eq}}, T) \tag{1} \]

The form given in Eq. (1) can easily be adapted to most computer codes since it uses variables already available in the codes. Two constitutive relations of this type have been proposed by Johnson and Cook [4] and Zerilli and Armstrong [5–7]. These two models are fundamentally different, as the first model is a phenomenological model, while the latter is based on dislocation theory. Both relations are known to be suitable for impact analysis in the sub-ordnance velocity regime, have relatively few material constants, are applicable for a variety of metals, and are considered easy to implement and use in computational procedures. Further, as these relations (and especially the Johnson–Cook (JC) model) seem to be the most used models for ballistic penetration studies, they have been chosen in this paper.

An extensive database exists on the high-strength steel alloy Weldox 460 E in terms of material tests as well as perforation tests [8–11]. Previous studies show that a modified version of the JC constitutive relation [8] combined with a modified version of the JC fracture criterion [8,12] can be calibrated using rather simple tensile tests [8,13]. Validation studies of this constitutive model (both the constitutive relation and the fracture criterion) show that the model gives good results on material test level [11,14]. Further, the constitutive model has been used in several numerical studies on the penetration and perforation of steel plates, and has also for these cases given reliable results [13,15,16].

However, the JC constitutive model has received some criticism over the years (see, e.g. [1]). This is partly due its empirical origin and partly due to the non-coupling between strain rate and temperature during plastic deformation. Thus, some consider the Zerilli–Armstrong (ZA) constitutive relation as a better choice in numerical simulations of impact related problems, but very little is found in the literature regarding this model associated to ballistic penetration. Nevertheless, some studies have compared these two constitutive relations in the literature. Harding [1] determined the material constants of the JC constitutive relation using torsion and tensile tests for three metals; OFHC copper, Armco iron and 4340 steel, respectively. The calibrated models were then used to simulate cylinders of the three metals being fired at rigid targets (Taylor tests). The results were compared with similar results obtained by Zerilli and Armstrong [5], and it was found that both models were in good agreement with the experimental results. However, the ZA relation gave a better prediction for OFHC copper. This is a similar conclusion as given in the paper by Zerilli and Armstrong [5]. In both papers it was assumed that the ZA model could be improved by making it more advanced and include other physical effects that was not yet accounted for (such as twinning). Harding further argued that despite these results, the constitutive relation may be considered a second-order effect compared with the boundary conditions of the problem studied.
Johnson and Holmquist [17] conducted a similar study on cylinder-impact tests for OFHC copper and Armco iron. They used the material constants already determined by Johnson and Cook [4] and by Zerilli and Armstrong [5] for the respective models. They came up with the same conclusion as Harding [1] and Zerilli and Armstrong [5], that both constitutive relations gave in general good agreement with the experimental data, but the ZA constitutive relation gave a better prediction of the final shape of the specimen. This work was later extended by the same authors using a modified version of the JC model and a coupled model combining the yield and strain hardening portion of the JC model with the temperature and strain-rate portion of the ZA model [18]. It was found that all four models show generally good agreement with cylinder impact experiments when using data generated from tension tests.

Macdougall and Harding [19] determined the material constants for the ZA constitutive relation for a Ti6Al4V alloy based on torsion tests performed over a wide range of strain rates on thin-walled tubular specimens. The calibrated constitutive relation was combined with a failure criterion based on a critical value of effective plastic strain, and the material model was then validated by simulating the same torsion tests that were used in the calibration. This clearly gave good agreement with the experimental results. The validated model was then used to predict the observed mechanical response in dynamic tensile tests performed on circular and rectangular cross-section specimens, and good agreement was obtained between the predicted and experimentally measured force–time curves.

Liang and Khan [3] reviewed and compared four exploited constitutive models for metals (i.e. Johnson–Cook, Zerilli–Armstrong, Bodner–Parton and Khan–Huang), and they were all found to be inadequate to describe the behaviour of tantalum where the work-hardening rate increased with increasing strain rate. Khan and Liang [20] studied further the behaviours of three bcc metals (tantalum, tantalum alloy with 2.5% tungsten and AerMet 100 steel) over a wide range of strain rates ($10^{-9}$–$10^{-7}$ s$^{-1}$) and temperatures (25–315°C). The JC and the ZA constitutive relation were demonstrated again to fail in describing experimental results for tantalum. A new constitutive model was proposed based on these experimental results. The proposed constitutive model gave good correlations with these experiments and strain-rate jump experiments. Khan et al. [21] then systematically studied the response of a Ti6Al4V alloy under quasi-static and dynamic loading, at different strain rates and temperatures obtained from monotonous and strain-rate-jump compression experiments. The JC constitutive relation were compared with a modified version of the proposed model by Khan and Liang [20], named as Khan–Huang–Liang (KHL) model. The KHL model is similar to the JC constitutive relation, but here the work-hardening is dependent on the strain rate so that decreasing work-hardening with increasing strain rate can be accommodated. Overall the KHL model correlated and predicted much closer to the observed responses than the JC constitutive relation for this titanium alloy as well as for two titanium alloys found in the literature.

Rohr et al. [22] demonstrated the necessity of a thorough determination of the model constants. They compared the JC constitutive relation and the ZA constitutive relation using the material parameters obtained for Armco iron. The material parameters were taken from the original references (i.e. Johnson and Cook [4] and Zerilli and Armstrong [5]). The comparison showed that there is a significant difference in the predicted stress–strain rate values for these two models at strain rates higher than $10^{3}$ s$^{-1}$, and thus there is a need for a thorough characterisation for all materials that are exposed to such high dynamic loading. The paper further involved characterisation of a high strength steel (35NiCrMoV109) over a wide range of strains, temperatures and strain rates, where the aim was to determine reliable material data and a material model for the numerical simulations of highly dynamic processes. Material data for very high strain rates ($10^{7}$ s$^{-1}$) were obtained, and it was found that for this particular steel the material behaviour at high strain rates could be better predicted by the ZA constitutive relation than by the JC constitutive relation. However, in a paper by Warren and Poormon [23] a rate sensitivity diagram was presented for VAR 4340 steel for strain rates up to $10^{8}$ s$^{-1}$ showing a linear behaviour between stress and strain rate. Hence, for this case it seems that the material behaviour can be better predicted by the JC constitutive relation than by the ZA constitutive relation.

All papers listed above compare the constitutive relations based on relatively simple material tests, and very few studies have been found in which the constitutive relations are evaluated on the basis of more complex tests on structural components. Meyer and Kleponis [24] used high strain rate strength data to determine the parameters for the JC and the ZA constitutive relation for the Ti6Al4V alloy. Due to lack of sufficient experimental data some of the parameters were taken from the literature. Both constitutive relations combined
with the fracture criterion by Johnson and Cook [12] were used to model a Ti6Al4V projectile penetrating a Ti6Al4V semi-infinite block at impact velocities up to 2000 m/s. Comparing these simulations with the experimental data the ZA constitutive relation was found to give good results, while the JC constitutive relation did not perform equally good. The authors however emphasised that these differences may lie in the used technique for determining the material constants, and believed that the results may improve with a more complete data set from which the parameters are determined.

Keeping previous work in mind, the objective of this study is to evaluate and compare the ZA constitutive relation with the modified JC constitutive relation. In the present study, the effect of constitutive relation for a typical penetration problem in the sub-ordnance velocity regime has been studied using the non-linear finite element code LS-DYNA [25]. The constitutive relations combined with the JC fracture criterion have been calibrated and validated for the target material Weldox 460 E steel using experimental data obtained from tensile tests where the effects of strain rate, temperature and stress triaxiality are taken into account [8,13]. Finally, these constitutive models are used in numerical simulations of projectile impact on steel plates, and the differences in results between the two models are discussed. It should be noted that a short version of this work was presented as a paper at the Computational Ballistics conference in Cordoba, Spain [26].

2. The material models

Weldox is a group of high-strength steels that combines high strength with high ductility and good weldability. The high strength in Weldox 460 E has been obtained by rolling at a particular temperature, followed by controlled cooling. The steel alloy consists of ferrite-pearlitic structure, where layers of pearlite lie between whole grains of ferrite. The pearlite makes the material highly ductile, while giving a reduction in strength. Fig. 1A shows the microstructure of Weldox 460 E, while Fig. 1B shows a typical stress–strain curve for a tensile test conducted on a smooth specimen under quasi-static condition at room temperature where both engineering and true values are given. One test series in Fig. 1B was conducted using an extensometer, while the other was conducted using a special device for measuring the diameter-reduction of the cross-section during testing [8]. From this latter series the true stress–strain curve could be obtained all the way to failure as seen in Fig. 1B. Further explanation about these measurements will be given in Section 3.2.
2.1. The Johnson–Cook (JC) constitutive relation

Johnson and Cook [4] proposed a phenomenological constitutive relation, which has been frequently used in impact analysis due to its simplicity. Here, the slightly modified version of the constitutive relation proposed by Børvik et al. [8] is used, and the equivalent stress is expressed as

\[ \sigma_{eq} = (A + B\varepsilon_{eq}^n)(1 + \varepsilon_{eq}^e C(1 - T^m)). \] (2)

The model operates with five material constants; \( A, B, C, n \) and \( m \). The dimensionless strain rate is given by \( \dot{\varepsilon}_{eq} = \dot{\varepsilon}_{eq}/\dot{\varepsilon}_0 \), where \( \dot{\varepsilon}_0 \) is a user-defined reference strain rate. The homologous temperature \( T^* \) is defined by \( T^* = (T - T_r)/(T_m - T_r) \), where the suffixes \( r \) and \( m \) indicate room and melting temperatures, respectively. The various phenomena such as strain hardening, strain-rate hardening and temperature softening are uncoupled in the constitutive relation, and the model is purely empirical. It should be noticed that Holmquist and Johnson also have proposed a modification of the strain-rate term in the original JC constitutive relation [18].

Two different versions of the modified JC constitutive relation have been proposed by the authors in the literature. In the original version by Børvik et al. [8], damage was coupled with the constitutive relation. In the coupled approach, material damage interacts with the stress–strain response. The advantage of the coupled approach is obvious — the softening effect due to material degradation is accounted for in the formulation. This softening may be important in order to capture strain localisation in shear bands prior to fracture. On the contrary, the stress and strain fields are assumed to be unaffected by damage in the uncoupled approach. Thus, the damage evolution can be calculated without any modification of the constitutive equations as in the original Johnson and Cook model, and damage can be computed as a part of the post-processing of data. Also, since the JC model accounts for thermal softening, it is possible to use the uncoupled approach to describe strain localisation in shear bands. The advantage of the uncoupled formulation is that the material constants can be obtained relatively easy. This, however, is at the expense of neglecting potential coupling of effects. Børvik et al. [15] used both the coupled and uncoupled approach of the modified JC constitutive relation in numerical simulations of plugging failure during ballistic penetration in order to investigate the importance of coupled damage. The numerical results were compared with observations from gas-gun experiments where blunt-nosed projectiles were launched against steel plates. It was found that the coupled model of viscoplasticity and ductile damage showed the best agreement with the experimental results in all simulations, but the differences between using a coupled and an uncoupled version of the modified JC constitutive relation was not significant. It was therefore, for simplicity, decided to use the uncoupled approach in this study.

2.2. The Zerilli–Armstrong (ZA) constitutive relation

While the modified JC constitutive relation is purely empirical, the ZA constitutive relation [5–7] is based on dislocation theory. Each material structure type has a different constitutive behaviour based on dislocation mechanics for that particular structure. The formulation is based on the idea of thermally activated motion of dislocations, and three constitutive relations have been developed. Depending on the structure, the models have six to eight material constants \( \sigma_a, A, B, n, \alpha_0, \alpha_1, \beta_0, \) and \( \beta_1 \). For bcc metals, the dislocation motion is governed mainly by the interaction with the overall lattice potential. The thermally activated motion is not dependent on the strain, so the strain hardening becomes independent of strain rate and temperature [5,6]

\[ \sigma_{eq} = \sigma_a + B \exp(-\beta T) + A\varepsilon_{eq}^n, \] (3)

where

\[ \beta = \beta_0 - \beta_1 \ln \dot{\varepsilon}_{eq}. \] (4)

For a fcc structure, the thermally activated motion is produced by dislocation intersections which are leading to strain-rate and temperature dependent strain hardening [5]

\[ \sigma_{eq} = \sigma_a + A\varepsilon_{eq}^n \exp(-\alpha T), \] (5)
where 
\[ \alpha = \alpha_0 - \alpha_1 \ln \dot{\varepsilon}_{eq} \]  
(6)
and \( n \) equals 1/2 in the original paper by Zerilli and Armstrong [5]. As hcp metals have a stress–strain behaviour falling somewhere in between bcc and fcc metals, the constitutive relation is described through a combination of the predominant interaction in the bcc structure and the predominant interaction in the fcc structure [7]
\[ \sigma_{eq} = \sigma_a + B \exp(-\beta T) + A \varepsilon_{eq}^n \exp(-\alpha T), \]  
(7)
where \( \beta \) and \( \alpha \) are as defined in Eqs. (4) and (6), respectively. This model is also applicable for steel alloys as well as hcp metals. By excluding some of the parameters one can either return to the original bcc model or the original fcc model. Since the ZA constitutive relation is theoretically based, it may seem reasonable to assume the ZA relation to be a better choice than the JC relation for this type of impact problems. However, the ZA constitutive relation is of a more complex form and involves coupling between the different effects, and the present study will show that this relation requires more experiments in order to determine the appropriate material constants compared with the JC constitutive relation.

2.3. The Johnson–Cook (JC) fracture criterion

Johnson and Cook [12] extended the fracture criterion by Hancock and Mackenzie [27] in order to make it dependent on strain path, strain rate and temperature, beside of being dependent on stress triaxiality. The fracture criterion is based on damage evolution, where the damage \( D \) of a material element is expressed as
\[ D = \sum \frac{\Delta \varepsilon_{eq}}{\varepsilon_f}. \]  
(8)
Here, \( \Delta \varepsilon_{eq} \) is the increment of accumulated (equivalent) plastic strain that occurs during an integration cycle and \( \varepsilon_f \) is the fracture strain. Failure is assumed to occur by element erosion when \( D \) equals unity, since no coupling between the damage and the constitutive relation is considered in this study. The fracture strain is constructed in a similar way as the JC constitutive relation. Given in a slightly modified version of the original model [8], it becomes
\[ \varepsilon_f = (D_1 + D_2 \exp(D_3 \sigma^*))(1 + \dot{\varepsilon}_{eq}^n D_4(1 + D_5 T^*)^D_6), \]  
(9)
where \( D_1, \ldots, D_5 \) are material constants. \( \sigma^* \) is the stress triaxiality ratio defined as \( \sigma_H/\sigma_{eq} \), where \( \sigma_H \) is the mean stress. As for the JC constitutive relation, the various phenomena accounted for in the fracture criterion are uncoupled. A more detailed study on the influence of the fracture criterion in projectile impact of steel plates has recently been submitted for publication [28].

In addition of using the modified JC fracture criterion in the numerical simulations, element erosion was also initiated when the homologous temperature reached a given critical temperature \( T^* = 0.9 \), indicating a maximum temperature of 1649 K (1376 °C) in the element before failure. This extra criterion was implemented in order to avoid numerical problems in severely distorted elements, which may occur in perforation problems. These destroyed elements do not add much structural strength to the model, and do consequently not affect the computation of the ballistic limit velocity.

3. Identification of material constants

Although the ZA constitutive relation is theoretically based, the required model constants are determined in a similar way as for the phenomenological models of Johnson and Cook. Hence, the determination of the material constants are performed through a best fit to specific material tests conducted for the high-strength steel Weldox 460 E [8,13]. Table 1 outlines the different tensile tests that were used in order to identify the parameters in each model.
3.1. The modified Johnson–Cook (JC) constitutive relation and fracture criterion

The calibration of the modified JC constitutive relation and the modified JC fracture criterion has been previously presented in Dey et al. [13], and only a brief summary will be given in the following. The calibrations for the JC models are based on an extensive series of tensile tests that were conducted on Weldox 460 E [8,13] (see Table 1). The stress-strain relations and fracture strains were obtained for a wide range of strain rates, temperatures and stress triaxiality levels using both smooth and pre-notched axisymmetric specimens. The parameters involving strain hardening, strain rate hardening, temperature softening and stress triaxiality were calibrated separately. Due to slightly different calibration methods by Dey et al.[13] and Børvik et al.[8,14], two sets of material constants for Weldox 460 E are given in the literature. The material constants given in Dey et al.[13] will be used in this study, and they are compiled in Table 2. It should be noted that the numerical results will depend on the chosen calibration method. In Dey et al. [28] a sensitivity study to determine in what way variations in the JC fracture model constants affected the ballistic limit velocity is shown.

In the original paper by Johnson and Cook [4], torsional material tests were also applied in the calibration of the material constants. Since torsion data are not available in this study, only standard tensile tests have been used to determine the material parameters. However, Holmquist and Johnson [18] showed that in cylindrical impact test, results obtained using tension data performed consistently better than those obtained using torsion/tension data. If this also yields for the perforation problem, dominated by localised shear, is still unknown.

3.2. The Zerilli–Armstrong constitutive relation

The two versions of the ZA constitutive relation that are considered in this study are the bcc model given in Eq. (3) and the hcp model (which is also applicable for steel alloys) given in Eq. (7). These will be noted as
Material parameters for the ZA constitutive relation

<table>
<thead>
<tr>
<th>Model</th>
<th>$\sigma_0$ (MPa)</th>
<th>$B$ (MPa)</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$A$ (MPa)</th>
<th>$n$</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZAbcc</td>
<td>11</td>
<td>388</td>
<td>$1.80\times10^{-4}$</td>
<td>$6.91\times10^{-5}$</td>
<td>553</td>
<td>0.2704</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>ZAhep*</td>
<td>20</td>
<td>378</td>
<td>$2.73\times10^{-4}$</td>
<td>$6.35\times10^{-5}$</td>
<td>563</td>
<td>0.2704</td>
<td>$-6.52\times10^{-5}$</td>
<td>$1.64\times10^{-5}$</td>
</tr>
<tr>
<td>ZAbcc (more data)</td>
<td>246</td>
<td>594</td>
<td>$5.19\times10^{-3}$</td>
<td>$2.13\times10^{-4}$</td>
<td>553</td>
<td>0.2704</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>ZAhep* (more data)</td>
<td>243</td>
<td>496</td>
<td>$4.65\times10^{-3}$</td>
<td>$1.88\times10^{-4}$</td>
<td>579</td>
<td>0.2704</td>
<td>$1.17\times10^{-4}$</td>
<td>$4.99\times10^{-6}$</td>
</tr>
</tbody>
</table>
temperature tensile tests. The stresses were however taken for small plastic strains—only up to 2.0%. Under the assumption that the strain hardening is negligible for these small values of plastic strains, the equation to be solved becomes

\[ \sigma_{eq} = \sigma_a + B \exp(-\beta_0 - \beta_1 \ln \dot{\varepsilon}_{eq}) T + A' \varepsilon_{eq}^{\sigma} \]  

(14)

under the constraint that

\[ \sigma_a = \sigma_a + B \exp(-\beta_0 - \beta_1 \ln \dot{\varepsilon}_{eq}) T \]  

(15)

when \( \dot{\varepsilon}_{eq} = 5 \times 10^{-4} \text{ s}^{-1} \) and \( T = 293 \text{ K} \). The ideal procedure would be to only use stresses corresponding to 0.2% plastic strain (initiation of yielding) so that the last term in Eq. (14) vanishes completely. These stresses were however difficult to determine for the highest strain rates \( 10^2-10^3 \text{ s}^{-1} \) in the dynamical tests (see Dey et al. [13]). Finally the remaining parameters \( A, x_0 \) and \( x_1 \) were determined using the true stresses for specific plastic strains in the higher regime of plastic strains (plastic strains of 4.0% up to 8.0% for most of the tests depending on initiation of necking). This time the entire model (Eq. (7)) was used, although with the constraint

\[ A' = A \exp(-x_0 - x_1 \ln \dot{\varepsilon}_{eq}) T \]  

(16)

when \( \dot{\varepsilon}_{eq} = 5 \times 10^{-4} \text{ s}^{-1} \) and \( T = 293 \text{ K} \). The obtained material constants are given in Table 3.

It is seen from Table 3 that the two ZA models have been calibrated to give two sets of material parameters. As mentioned, there is a coupling between the effects of strain rate and temperature in the two ZA models that may need to be considered in the calibration. Børvik et al. [11] performed material tests on Weldox 460 E in a Split Hopkinson Tension Bar where the initial temperature varied between 100 and 500 \( ^\circ \text{C} \). The combined effects of high strain rate, elevated temperature and stress triaxiality were studied by testing both smooth and pre-notched specimens. It was found that the temperature dependence of the stress–strain behaviour differs at high strain rates compared with quasi-static conditions. The material data from the tests of Børvik et al. [11] involving coupling between strain rate and temperature were therefore also taken into account in the calibration of the latter material set in Table 3 (noted as ‘more data’). Table 1 reveals the different types of tensile tests used in the calibration. Note that this extended data range will not affect the strain hardening behaviour under quasi-static condition.

Fig. 2 compares the calibrated constitutive relations with the experimental data that were used in the calibration procedure. It is seen in Fig. 2A that the strain hardening of the material under quasi-static conditions is well described by the different relations. Further, the JC constitutive relation gives satisfactory results within the calibrated range for both temperature softening and strain rate hardening as seen in Fig. 2B and Fig. 2C, respectively. It is further seen that the two ZA models describe the strain rate hardening with satisfaction, while the temperature softening is described poorly. Moreover, minor differences are seen

Fig. 2. Comparison between experimental data and the calibrated constitutive relations for the (A) strain hardening, (B) temperature softening and for (C) strain rate hardening.
between the two ZA models within the calibrated range. As the material only has been tested up to 500 °C and
the strain rate has a maximum value of about $10^3$ s$^{-1}$, the calibration is solely based on this range of data.
Nevertheless, the calibration is assumed to also be valid for higher temperature conditions as well as higher
rates of strains. Fig. 2B and C show that there are larger deviations between the models when the temperature
and the strain rate exceed the test range used in the calibration of the models. As also observed by Rohr et al.
[22], the JC constitutive relation seems to give more temperature softening and less strain rate hardening than
the ZA models. Further, when the ZA models are calibrated using the material data involving coupling
between strain rate and temperature (noted as ‘more data’), the models give less temperature softening and
more increase in strain rate hardening than if this data is not accounted for. The validation study in the
following section will show the consequences of these deviations.

4. Comparison and validation of the material models

4.1. Experimental background

In order to validate the different material models in Section 2, numerical simulations of the tensile tests
conducted by Børvik et al. [11] are performed. As previously mentioned, these are tensile tests conducted
under the combination of dynamic loading, elevated temperature and stress triaxiality using a Split Hopkinson
Tension Bar. It should be kept in mind that none of these tests were used in the calibration of the JC
constitutive relation, while only data from smooth specimen tests from this particular study [11] were used in
the calibration of the ZA models (noted as ‘more data’ in Table 3). The geometry and dimensions of the
specimens that were used in the tests are shown in Fig. 3. Three temperatures $T_0$ were considered; 100, 300 and
500 °C. The average engineering strain rates $\dot{\varepsilon}_{\text{avg}}$ were between 450 and 550 s$^{-1}$ in the smooth specimens. Three
notch radii ($R_0$) were chosen; 0.4, 0.8 and 2.0 mm in addition to smooth specimens. It is referred to the original
paper by Børvik et al. [11] for further details about the experimental programme carried out.

4.2. Numerical set-up

The different constitutive models presented in the previous section were implemented as user-defined
material models in the explicit non-linear FEM code LS-DYNA [25]. Since the constitutive models are used
for adiabatic conditions, it is necessary to estimate the increase in temperature caused by dissipation of plastic

\[ E (GPa) \quad \nu \quad \rho (kg/m^3) \quad C_p (J/kgK) \quad \chi \quad \alpha (K^{-1}) \quad \sigma_0 (MPa) \quad E_t (MPa) \]

| Material tests/target | 210 | 0.33 | 7850 | 452 | 0.9 | $1.2 \times 10^{-5}$ | — | — |
| Projectile           | 204 | 0.33 | 7850 | —   | —   | 1900            | 15000 |

Fig. 3. Geometry and dimensions of smooth and notched test specimens for high rate tests (mm). It should be noted that for
manufacturing reasons, the side faces of the notch were inclined at an angle $\alpha = 17.5^\circ$ to the normal of the specimen axis for specimens
with $R_0 = 0.4$ and 0.8 mm, while for $R_0 = 2.0$ mm the side faces were normal to the specimen axis [14].
work. The temperature increment due to adiabatic heating is calculated as [31]

$$\Delta T = \int_0^{\text{eq}} \chi \frac{\sigma_{\text{eq}} \, d\varepsilon_{\text{eq}}}{\rho \, C_p},$$

where $\rho$ is the material density, $C_p$ is the specific heat, and $\chi$ is the Taylor–Quinney coefficient that represents the proportion of plastic work converted into heat and is usually assigned 90% for metals [32].

For this study the ZAbcc model and the ZAhcp* model will have two sets of material parameters; one based on tensile tests with no coupled effects, and one where the coupled effect between strain rate and temperature is accounted for by using the smooth specimen tests from Børvik et al. [11]. The calibration of the modified JC constitutive relation and the modified JC fracture criterion are solely based on the tensile tests with no coupled effects [8,13]. The material constants for the different models are given in Tables 2 and 3, while other relevant material constants are given in Table 4.

Both smooth and pre-notched axisymmetric specimens with mesh geometry as shown in Figs. 3 and 4 were analysed. The notched specimens have initial notch root radius $R_0$ equal to the experimental values; 0.4, 0.8 and 2.0 mm, while the initial minimum radius $a_0$ was 1.5 mm in all models. The specimens were modelled using four-node axisymmetric elements with one integration point and stiffness-based hourglass control [25]. The study by Børvik et al. [14] indicates that 20 elements across the specimen radius was a reasonable choice both regarding numerical accuracy and computational efficiency. This gave an element size in the critical gauge region of approximately $0.075 \times 0.075 \text{ mm}^2$, which resulted in 3400 elements (3591 nodes) for the smooth specimens, while 4560 elements (4809 nodes) were used for notched specimens.

The loading was defined by describing the elongation at one end of the specimen against time, using the measured elongation–time curve from typical experiments, while the other end was fixed. Since only the specimen, without the bars, is modelled in the numerical simulations, it follows that the calculated force is disturbed by an elastic wave that propagates along the specimen length and reflects from the specimen ends. Hence, severe oscillations occurred in some of the numerical force–time curves, while this was not observed in the experimental curves. The wavelength of the oscillations agrees well with the time it takes for the elastic wave to travel twice from one side of the specimen to the other. In order to reduce the force oscillations, the force signals from the numerical simulations were filtered using a cosine filter with a cut-off frequency of 10,000 Hz. The filtering procedure is described in more detail in Børvik et al. [14].

4.3. Comparing results

The force–elongation curves from the simulations are compared with the experimental results of Børvik et al. [11] in Fig. 5. Firstly, it is seen that the modified JC constitutive model (both constitutive relation and fracture criterion) is in reasonably good agreement with the experimental results both for the maximum force

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Fig. 4. Axisymmetric finite element meshes for smooth and notched specimens.
Fig. 5. Experimental versus numerical force–elongation curves for smooth and notched specimens at temperatures 100, 300 and 500 °C.
as well as the elongation to fracture. The largest deviations are found for tests at 500 °C. Secondly, it is easily observed that both the ZAbcc model and the ZAhcp* model deviate severely from the experimental results when the calibration is only based on data where the effects of strain rate and temperature are uncoupled. On the other hand, it is seen that both these models predict the behaviour well when the coupled data on strain rate and temperature are included. These results show how essential data for the coupled influence of strain rate and temperature are when calibrating the ZA models. Further it is seen that there are almost no differences in the behaviour between the bcc model and the hcp* model. The prediction of the maximum force is especially in good agreement with the experimental data, but also the elongation to fracture is well predicted for most cases. The largest deviations are again seen when the temperature is 500 °C, but some deviation is also seen for the smooth specimens at 300 °C. These deviations are most likely due to the strain ageing effect called blue brittleness [33]. This phenomenon is not properly accounted for in any of the models. The blue brittle region occurs around 200–300 °C when the strain rate is low (quasi-static condition), but this temperature region seems to move with strain rate [34]. Another observation is that the ZA models in general show less temperature softening than the modified JC constitutive relation (see also Fig. 2).

Again, it should be emphasised that the experimental results presented in this section were not used in the identification of the constitutive relation and the fracture criterion of Johnson and Cook. The only experimental results that were used are the results from the smooth tests shown in Fig. 5 for identifying the parameters for the two ZA models (noted as ‘more data’ in Table 3). As the results indicate, this was necessary in order for the models to be in better agreement with the experimental results. Overall it is seen from the plots that the main trends for smooth and notched specimens at high strain rates and elevated temperature are reasonably well captured by the numerical simulations. It may also be concluded that the differences in results obtained between the simulations using the JC constitutive relation and the ZA constitutive relation are not that severe. For calibration purposes, however, the JC constitutive relation requires less and simpler tensile tests than the ZA constitutive relation in order to predict similar material behaviour. As a final comment, it should be noted that elements in the validation study were eroded only due to the JC fracture criterion and not due to the temperature criterion since the homologous temperature \( T^* \) will never reach the critical value in these simulations. Hence, elements are only eroded when the damage parameter \( D \) reaches unity. The stresses in the element are then set to zero and the element is removed from the mesh.

5. Perforation of steel plates

5.1. Component tests

The perforation tests on Weldox 460 E steel plates with blunt projectiles [10] were conducted in a compressed gas gun, where the sabot-mounted projectiles were launched at impact velocities just below and well above the critical impact velocity, i.e. the ballistic limit velocity, of the target plate. The sabot pieces were stopped by a sabot trap so that the projectile could travel freely before it impacted the target after about 2 m of free flight. All variables except for the impact velocity were kept constant in the tests. The targets, having a nominal thickness of 12 mm and a free span diameter of 500 mm, were clamped in a rigid frame. Further, the projectiles were manufactured from Arne tool steel, having a nominal Rockwell C hardness of 52 (\( \sigma_0 \approx 1900 \text{ MPa} \)) after hardening, and nominal mass, diameter and length of 197 g, 20 mm and 80 mm, respectively.

Initial and residual velocities were obtained from various optical measurement systems. In addition, the perforation process was captured using a digital high-speed camera system. The digital images made it possible to measure impact angles and projectile velocity during perforation. Details regarding the test set-up may be found in Børvik et al. [10].

Some data from the experimental tests are given in Table 5, and the notation used can be found in the Nomenclature. Besides of these data, the projectile pitch and target oblique were obtained and found to be small. From the measured initial velocities the ballistic limit velocity of the target was estimated to be 184.5 m/s, given as the average between the highest impact velocity not giving perforation and the lowest impact velocity giving complete perforation. High-speed camera images of test #16 will be compared with numerical simulations in Section 5.3. The experimental tests show that perforation occurs by plugging. For this failure
mode the deformation localises in narrow shear bands where shear strain, shear strain rate and temperature may locally be very high. A close-up image of the cross-section of test #9, where perforation did not occur, is shown in Fig. 6A. The projectile indentation and shear localisation that occurs during impact are clearly seen in the image. The residual velocity \( v_r \) versus the initial velocity \( v_i \) curves are given for both the projectile and plug in Fig. 6B. Here, the solid lines through the data points are curve fits of the analytical model by Recht and Ipson [35]

\[
v_r = a (v_i^p - v_{i0}^p)^{1/p}.
\]  

### Table 5
Experimental data for 12 mm thick Weldox 460 E steel plates [10,37]

<table>
<thead>
<tr>
<th>Test #</th>
<th>( v_i ) (m/s)</th>
<th>( v_r ) (m/s)</th>
<th>( v_{rpl} ) (m/s)</th>
<th>( m_{pl} ) (g)</th>
<th>( w_{max} ) (mm)</th>
<th>( d_{fr}/d_{tre} ) (mm)</th>
<th>( h_{tc} ) (mm)</th>
<th>( h_{pl} ) (mm)</th>
<th>( d_{fr}/d_{tre} ) (mm)</th>
<th>( d_{pl} ) (mm)</th>
<th>( w_{pl} ) (mm)</th>
<th>( D_L ) (mm)</th>
<th>( t_f ) (μs)</th>
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<td>242.3</td>
<td>74.3</td>
<td>19.0</td>
<td>20.7/20.9</td>
<td>16.2</td>
<td>11.5</td>
<td>19.0/20.9</td>
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<td>1.01</td>
<td>0.90</td>
<td>60</td>
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<td>03</td>
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<td>181.1</td>
<td>224.7</td>
<td>71.7</td>
<td>19.6</td>
<td>20.7/20.8</td>
<td>15.6</td>
<td>11.4</td>
<td>20.7/21.7</td>
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<td>65</td>
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<td>187.7</td>
<td>66.9</td>
<td>21.6</td>
<td>20.4/21.2</td>
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<td>0.28</td>
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<td>47.4</td>
<td>21.9</td>
<td>20.4/20.6</td>
<td>14.7</td>
<td>11.7</td>
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<td>0.0</td>
<td>2.93</td>
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<td>4.6/-</td>
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<td>2.0</td>
<td>4.6/-</td>
<td>0.0</td>
<td>0.14</td>
<td>0.22</td>
<td>10</td>
</tr>
</tbody>
</table>

\(^a\)Estimated from high-speed camera images.

\(^b\)Measurements not available.

\(^c\)Projectile nose broke at impact.

\(^d\)No image available.

\(^e\)Projectile indentation.

Fig. 6. (A) Cross-section macrograph of test #9 showing projectile indentation and shear localisation and (B) experimental data as residual velocity versus initial velocity for both the projectile and plug.
The Recht–Ipson model is based on the conservation laws, so that in the original model $a = m_p / (m_p + m_{pl})$, where $m_p$ and $m_{pl}$ are the mass of the projectile and plug, respectively, $p = 2$ and $v_{bl}$ is the ballistic limit velocity. Here, the model constants $a$ and $p$ were fitted to the experimental results using the method of least squares, and $v_{bl}$ is determined experimentally. Fig. 6B shows that the residual velocity of the plug is always found to be higher than the residual velocity of the projectile. The sudden increase in plug velocity after fracture is believed to be caused by elastic stress waves propagating back and forth in the projectile and the plug [10]. Some typical cross-sections of penetrated target plates are shown for increasing impact velocity in Fig. 7. It is seen that the deformation is highly localised regardless of the impact velocity, but the amount of energy absorbed in global target deformation increases as the impact velocity approaches the ballistic limit.

Table 5 shows that the maximum deformation ($w_{max}$) for perforated plates increases when the impact velocity is close to ballistic limit velocity, but is only about 1/4 of the target thickness at the most. More details about the experimental results can be found in Børvik et al. [10].

5.2. Numerical models

In Section 4.3 it was seen that for material tests the JC constitutive relation and the ZA constitutive relation were found to give reasonable and similar results when properly calibrated. A natural continuation of this study would be to find out whether the same prediction can be made for a structural component. The impact tests for blunt projectiles presented in Section 5.1 were therefore analysed using LS-DYNA. The target was modelled using the same material models as presented in Section 2. Hence, only the material model of the target plate varied in the simulations due to different constitutive relations. Based on the validation study, it was seen that it was essential to take the coupled effect of temperature and strain rate into account during calibration. It is therefore expected that the ZA models will predict the perforation process poorly if this coupled effect of temperature and strain rate is not included. Hence, only the material parameters for the ZA models noted as ‘more data’ in Table 3 will be used in the following. The material parameters for the JC constitutive relation and the JC fracture criterion are as given in Table 2. The projectile of hardened tool steel was modelled as a bilinear elastic–plastic von Mises material with isotropic hardening without fracture using Material Type 3 in LS-DYNA [25]. The material constants for the projectile together with other relevant material constants needed for the target are given in Table 4.

The geometry of the target and projectile was identical to that used in the experimental tests. The circular target plates with nominal thickness of 12 mm and free diameter of 500 mm were fully clamped at the support. The projectiles had a nominal mass, diameter and length of 197 g, 20 mm and 80 mm, respectively. Again the problem to be solved was axisymmetric, and 2D analyses could be performed. The same type of elements was used as in the validation study in Section 4.2. Thus, the mesh was fixed using four-node axisymmetric 2D elements with one integration point and stiffness-based hourglass control. The contact between the projectile and the target was modelled using an automatic 2D single surface penalty formulation without friction available in LS-DYNA [25]. Frictional effects are assumed to be of minor importance in this particular problem [2,15].

The initial configuration of the target and projectile are given in Fig. 8. Due to the use of axisymmetric elements, Fig. 8 shows the model with a reflected part. The target consisted of two parts with identical properties; one local and one global part. The local part was slightly larger than the diameter of the projectile.
(0.5 mm larger on each side, i.e. indicating a total size of 21 mm, where only 10.5 mm was meshed). In order to reduce the computational time, which is affected both by the element size and the total number of elements, the mesh in the global part was coarsened towards the boundary. The mesh density for the target given in Fig. 8 is 60 elements over the target thickness. For this mesh density, the initial size of the smallest element in the local region is $0.25 \times 0.20 \text{ mm}^2$, and the total number of elements in the target plate is 9240. A much coarser mesh was chosen for the projectile, which had an initial element size of $1.0 \times 1.0 \text{ mm}^2$ indicating a total number of 800 elements.

Perforation problems with blunt projectiles involving shear localisation and plugging are highly sensitive to mesh density. In a mesh-sensitivity study by Børvik et al. [15] it was found that the numerical simulations seemed to converge monotonically towards a limit solution when the number of elements over the target thickness becomes sufficiently large. Hence, the sensitivity was not found to be pathological as long as a viscoplastic constitutive relation was applied. In that study it was further concluded that due to computational limitations it was too time consuming and therefore unpractical to have finer mesh densities than 120 elements over the target thickness in order to obtain the ballistic limit velocity. Further, Dey [36] showed that by increasing the number of elements from 120 to 240 over the target thickness, the ballistic limit velocity was only slightly decreased. Thus, for most practical applications, it may be reasonable to assume that the solution has converged for this particular problem using 120 elements over the target thickness even though it is known that the width of a typical shear band in the target is less than the size of the smallest element [13]. However, since it is not clear whether or not the solution is fully converged, this problem requires further studies.

The study by Børvik et al. [15] only considered the JC constitutive relation. Here, three different element densities were considered to find out in what way the choice of constitutive relation will affect the mesh dependency. These densities were 30, 60 and 120 elements over the target thickness. For 120 elements over the target thickness the initial size of the smallest element is $0.100 \times 0.125 \text{ mm}^2$, and the total number of elements in the target is 29640.

5.3. Numerical simulations

In the simulations, the projectile was given an initial velocity similar to the one used in a corresponding experiment and the residual velocity of the projectile was registered. From this, the ballistic limit velocity $v_{bl}$ was estimated based on 6-7 runs using the analytical model by Recht-Ipson. Now the method of least squares was used to fit all three model constants ($a$, $p$ and $v_{bl}$) to the simulated residual projectile velocities. These values are given in Table 6 using 120 elements over the target thickness, where also the values obtained from the experiments are given for comparison. Note that the model constants obtained from the simulations where the JC constitutive relation was used differ slightly from what has been presented previously in Dey et al. [36]. This is because some additional simulations have been included in this study. The impact velocities with the

Fig. 8. Plot of initial configuration of the three different parts; the blunt projectile, the local region and only a small part of the global region of the target plate.
obtained residual velocities are given in Table 7 along with some other important data from the numerical simulations. Fig. 9A shows the residual velocity versus impact velocity curves from simulations using the finest mesh (i.e., 120 elements over the target thickness) where the solid lines are the Recht–Ipson model. Also the experimental results are included in the figure for comparison. It is seen that the modified JC constitutive relation predicts the ballistic limit velocity considerably better than the ZA constitutive relation. Further, the two versions of the ZA constitutive relation predict almost the same ballistic limit velocity and $v_r - v_i$ curve.

Although there is a considerably difference in the ballistic limit velocity for the modified JC constitutive relation and the ZA constitutive relation, the curves coincide at around 400 m/s. It is therefore concluded that the differences in the material models are most prominent at impact velocities close to the ballistic limit. Fig. 9B confirms the previous observations by Børvik et al. [15] that the ballistic limit velocity is highly dependent on the mesh density when the modified JC constitutive relation is used. It is further seen that the mesh size dependency seems to be less distinct for the two versions of the ZA constitutive relation. This may be caused by the stronger strain-rate sensitivity predicted by the ZA constitutive relation than by the JC constitutive relation. Hence, the choice of constitutive relation affects the mesh size dependency. In addition, the two ZA models predict almost the same ballistic limit velocity except for when 60 elements over the target thickness is used.

Fig. 10 shows high-speed camera images of test #16, which is compared with fringe plots of a corresponding numerical simulation for each constitutive relation. The mesh density was 120 elements over the target thickness.

### Table 7
Numerical data from simulations using 120 elements over the target thickness

<table>
<thead>
<tr>
<th>Model</th>
<th>$v_i$ (m/s)</th>
<th>$v_f$ (m/s)</th>
<th>$v_{ipt}$ (m/s)</th>
<th>$w_{max}$ (mm)</th>
<th>$T_{max}$ (K)</th>
<th>$E_{L,D}$, $E_{L,T}$ (µs)</th>
<th>$t_f$ (µs)</th>
<th>Run time (µs)</th>
<th>CPU (h.m.s)</th>
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</tr>
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<td>170.0</td>
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<td>ZAhcp*</td>
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<td>175.0</td>
<td>170.0</td>
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<td>155.0</td>
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| Experimental & Numerically obtained ballistic limit velocities and Recht–Ipson constants. The numerical results are for 120 elements over the target thickness

Table 6
Experimental Johnson–Cook Zerilli–Armstrong - bcc Zerilli–Armstrong - hcp*

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<th>$p$</th>
<th>$v_{th}$ (m/s)</th>
<th>$a$</th>
<th>$p$</th>
<th>$v_{th}$ (m/s)</th>
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<th>$p$</th>
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</tbody>
</table>
thickness, and the plots show fringes of effective plastic strain where the darkest areas indicate a plastic strain above 2. All series shown are taken at impact velocities 1–3% above the respective ballistic limits. It is seen that the simulations can predict the correct failure mode, and that the deformation mainly develops in a narrow, almost horizontal zone in the target. Hence, the overall physical behaviour of the structure seems to be well captured for all constitutive relations. However, further investigations show that the simulation where the JC constitutive relation is used has a stronger localisation and less global bending than when the ZA constitutive relation is used. Comparing the maximum target deformation \( w_{\text{max}} \) for each simulation with test #16, it can be seen from Tables 5 and 7 that the JC, ZAbcc and ZAhcp* models give around 2, 3 and 3.5 times higher values of \( w_{\text{max}} \) than the experimental value, respectively. Note that in the numerical simulations both the elastic and the plastic deformations are present. These values are somewhat higher than those observed by Børvik et al. [10], but in their numerical simulations, the JC constitutive relation was coupled with ductile damage mechanics. Furthermore, around 20% more elements fail in the analysis when using the ZAbcc model compared with the JC constitutive relation, while around 50% more elements fail in the analysis when using the ZAhcp* model, again compared with the JC constitutive relation. Fig. 11 shows the fracture pattern for the same simulations shown in Fig. 10. It is seen that all three material models give a distinct crack along the shear zone, but the JC constitutive relation gives a cleaner cut than the other two. The width of the shear band for especially the ZAhcp* model is wider than for the JC constitutive relation, since more elements fail in the former. Furthermore, it is found that at impact velocities close to the ballistic limit velocity the direction of the fracture propagation differs between the two models. For the JC constitutive relation the fracture mainly propagates from the front side towards the rear side of the target, which is in accordance with experiments. On the contrary, for both versions of the ZA constitutive relation fracture mainly propagates from the rear side towards the front side of the target. Thus, the ZA models does not predict the correct fracture mode. This was however not seen for the highest impact velocity (400 m/s). Here, all models predict the fracture to propagate from the front side towards the rear side of the target.

As discussed in relation with Fig. 9, the material models predict almost the same residual velocity when the impact velocity is much higher than the ballistic limit. Fig. 12 shows results from numerical simulations for each material model with impact velocity equal to 400 m/s. It is seen that for this case all material models predict the perforation process with similar results. The projectile becomes more deformed compared with a lower impact velocity, which seems reasonable. From these fringe plots as well as from the fracture pattern of these simulations shown in Fig. 13, it can be seen that the total number of failed elements is higher than for runs with lower impact velocity. Further, the patterns become more diffuse and two propagating cracks occur, which implies that a small part detaches from the target in addition to the plug. This fragment is also visible in Fig. 12, but was not observed in any of the experiments. Some differences between the simulations can however be detected. Again the maximum target deformation \( w_{\text{max}} \) is better predicted using the JC...
constitutive relation than using the two ZA models when compared with test #20 (see Tables 5 and 7). This is a general observation between these two constitutive relations regardless of impact velocity. The number of failed elements is higher when using the JC constitutive relation compared with the two ZA models, which is opposite of what is seen for the lower impact velocities (see Table 7). In addition of using the modified JC fracture criterion, element erosion was as mentioned initiated when the temperature in the element reached 1649 K (1376 °C). It is seen from Table 7 that very few elements erode due to this temperature criterion compared with the JC fracture criterion. However, for the two ZA models relatively more elements seems to fail due to the temperature criterion. The time of failure ($t_f$) is in general lower for the JC constitutive relation than for the two ZA models, and is closer to the experimental data. All these differences may be the result of how the constitutive relations describe the temperature softening and strain rate hardening. With reference to

Fig. 10. High-speed camera images of an experimental test compared with numerical analyses at impact velocities close to the respective ballistic limits. The target plotted as fringes of effective plastic strain in the user-defined range $e_{eq} = 0$ (light grey) and $e_{eq} = 2$ (black).
Fig. 2, it was seen that the JC constitutive relation gives more temperature softening and less strain rate hardening than for the two ZA models. Hence, the number of failed elements and time of failure will be affected. It is seen that the values of these three measures are higher for the ZA models compared with the JC constitutive relation. In general, small deviations are seen between the two ZA models, while the JC constitutive relation seems to give better predictions than the ZA constitutive relation of the investigated impact problem.

Fig. 11. Removed elements giving fracture patterns for each material model at impact velocities just above each respective ballistic limit velocity.

Fig. 12. Numerical analyses of perforation test at $v_i = 400$ m/s. The target plotted as fringes of effective plastic strain in the user-defined range $e_{eq} = 0$ (light grey) and $e_{eq} = 2$ (black).
6. Concluding remarks

The overall conclusion of this study is that although both constitutive relations predict the physical mechanism of the perforation process well, the modified JC constitutive relation seems to quantitatively predict the ballistic limit velocity better than the ZA constitutive relation. This is interesting, since it was found that numerical simulations of tensile tests under various loading conditions predict the experimental results with reasonably good agreement for both the JC and the ZA constitutive relation. On the other hand, at very high temperatures and high strain rates, there is a significant difference in the predicted temperature softening and the strain rate hardening between the JC constitutive relation and the ZA constitutive relation. Such high strain rates and temperatures appear in ballistic penetration problems. For calibration purposes, the JC model requires less and simpler tensile tests than the ZA constitutive relation in order to predict the same material behaviour. Based on the observations from this study and under the conditions investigated, it may from an engineering point of view seem reasonable to use the modified JC constitutive relation in simulations of projectile impact on steel plates instead of a more sophisticated constitutive relation like the ZA relation.

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References


