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SENSITIVITY OF PROJECTILE MUZZLE RESPONSES TO GUN BARREL CENTERLINE VARIATIONS

Michael M Chen

US Army Research Laboratory, AMSRD-ARL-WM-BC, APG, MD 21005-5066, USA

Gun barrel centerline variations have been known to impact projectile performance. The centerline variations were modeled with four shape variables. The range of each shape variable was determined based on the characteristics of a 70-caliber smooth gun tube. This study focused on how the muzzle responses, including three velocity components and yaw and pitch angles of a 60-mm projectile, were influenced with the shape variables. Design of experiments technique was adopted to constitute an array of design cases with distinctive barrel shapes. The muzzle responses of each case were obtained individually in parallel. Subsequently, analysis of variance method was employed to identify the sources of response variability and to determine the importance of the shape variables. The main effect of each controlled factor was studied and their interactions contributing to the muzzle responses were also outlined. In addition, an approximate regression model was derived for each response and the goodness of fit was discussed.

INTRODUCTION

Muzzle velocity and angle of attack are two of the most important indicators for gun-projectile performance. Nevertheless, due to a variety of uncertainties, it has not been easy to obtain repeatable muzzle responses from experimental tests. As a result, understanding the contributing factors to the variations of muzzle responses is essential when determining the reliability of an ammunition system. In the past decade, a few researchers have conducted studies to identify and quantify variables that may affect muzzle velocity [1-3]. In previous investigations, a total of 17 random variables including their initial conditions were identified and utilized in the studies. Although the interior ballistic modeling in the literature was quite comprehensive, it did not cover gun barrel centerline variations that have been known to significantly influence in-bore projectile behavior. Thus, Bundy et al. [4] assessed the interactions between tank motion and gun barrel rotation and translation by enumerating 10 most likely barrel shape combinations for the evaluation of gun accuracy. In addition, Erline [5] analyzed a number of hypothetical cases where a gun barrel centerline changed from a bent state to an unbent state, and found that lateral loads could be dramatically amplified by a small sine wave in a gun barrel centerline. This paper employed design of experiments

(DOE) techniques to simulate a large number of barrel shapes and investigated the sensitivity of muzzle responses to the barrel shapes.

A 60-mm projectile system that consisted of a penetrator, projectile body, electronics, sabot, and obturator, was utilized for the study. The penetrator was made of tungsten material, body wall of steel, sabot of aluminum, and obturator of nylon. The in-bore structural dynamic analysis of the projectile system except optimized sabot has been previously performed [6]. For computational efficiency in DOE analysis, the windscreen and stabilized fins were substituted with equivalent weight such that the center of gravity of the projectile system remained at the same location. The projectile configuration and grids are displayed in Figure 1. This model contained solely hexahedral elements with a total number of 42,984. The entire mass of the projectile system including sabot was approximately 1 kg. A 70-caliber smooth gun tube was used and the total in-bore travel distance for the projectile was 3,840 mm. Given 1.3 liter gun chamber and M2 propellants with geometry of 7-perforation grain, IBHVG2 yielded a base pressure curve as shown in Figure 2. A peak pressure of 315 MPa took place at 2 ms after ignition and the total in-bore travel time was 4.7 ms. Since the primary focus of the study was the effects of barrel flexure on muzzle responses, the rigid body responses of the projectile at the exit were used for the comparison.

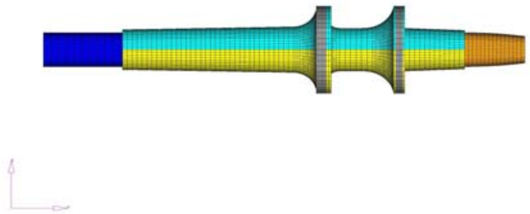


Figure 1. Configuration of a 60-mm projectile system

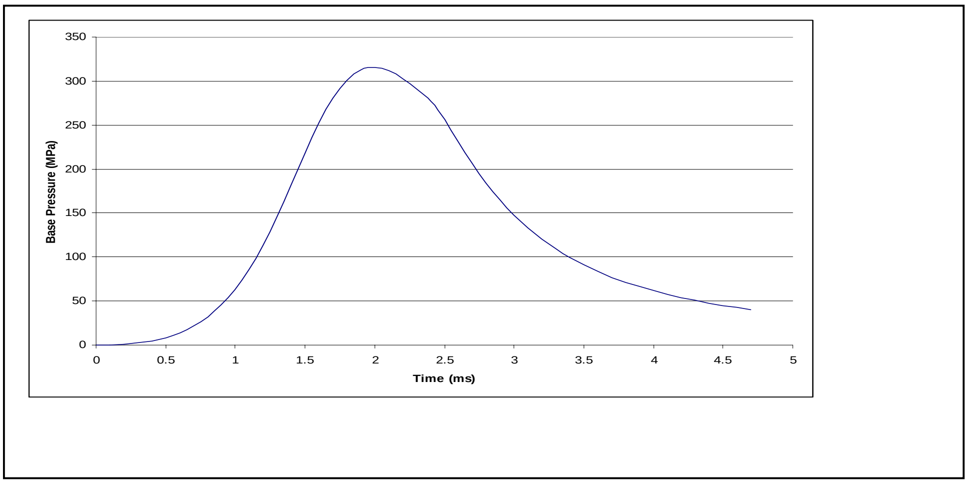
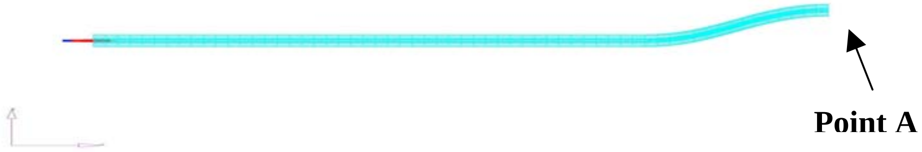


Figure 2 Time history of base pressure

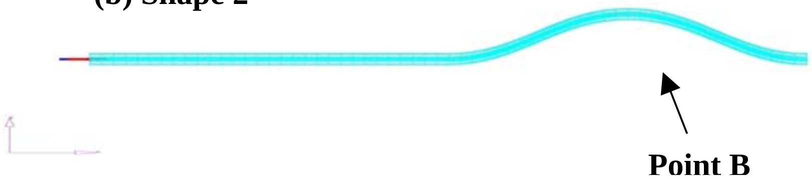
DESIGN OF EXPERIMENTS

The centerline variations of a gun barrel may be attributed to a number of factors, such as manufacturing errors, uneven cooling, non-uniform wall thickness/erosion, vibrations, etc. Regardless of the sources of the variations, four fundamental shapes were created and given in Figure 3. Any combination of these four shapes may account for the centerline variations due to one or more of the causes. Note that the displayed shapes have been magnified for visibility. In consideration of the flexed barrel profile from a total of 37 cases concluded by Bundy et al., the choice of the four basis shapes should constitute a

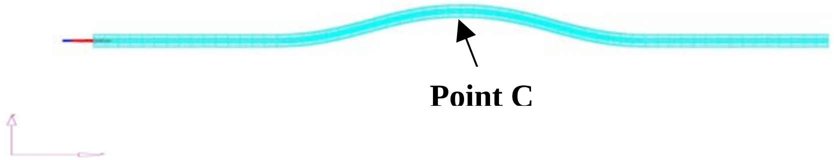
(a) Shape 1



(b) Shape 2



(c) Shape 3



(d) Shape 4

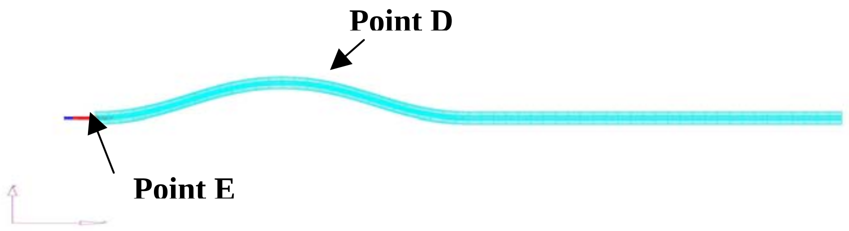


Figure 3. Display of four exaggerated deflection shapes of a gun barrel

good representation of the profile. The steps to generate the barrel shapes can be described as follows: First, the gun barrel was equally divided into four areas and five points named A (muzzle), B, C, D and E (breech); Second, point E was completely constrained and the other four points were to be transformed only laterally (Y direction); Third, point A was first displaced a unit while the other points were fixed, i.e. shape 1; note that a second order biasing factor was used for the morphing such that the continuity of the slope could hold; and finally the third step was repeated for the other three points B, C and D, which led to the generation of shapes 2, 3 and 4, respectively.

The ranges of absolute lateral displacements along down-bore distance from rear face of the gun tube for these five points are summarized in Table 1 in which the numbers in the parenthesis represent the ratio to the total length of the barrel. The magnitude of the displacements was determined based on the characteristics of ten most likely barrel shapes proposed by Bundy et al. Figure 4 demonstrates a typical gun barrel centerline. In DOE analysis, because the locations of points A and B exhibit higher deviations in the profile, a total of 5 levels was selected as opposed to only 3 levels for points C and D. As mentioned, point E was completely fixed at all time. To obtain a whole spectrum of barrel shapes, a full factorial design was selected. As a result, a total of 225 (5x5x3x3) design cases were generated for the study. Because each case could be solved individually, thanks to high performance computing resources available at U.S. Army Research Lab, the elapsed time for the total computations was less than 24 hours.

Table 1 Absolute lateral displacement (mm) of a gun barrel at five chosen points

	Point E	Point D	Point C	Point B	Point A
Level 1	0	0.03 (0.0008%)	0.05 (0.0013%)	0.21 (0.0055%)	0.25 (0.0065%)
Level 2		0	0	0.105	0.125
Level 3		-0.03	-0.05	0	0
Level 4				-0.105	-0.125
Level 5				-0.21	-0.25

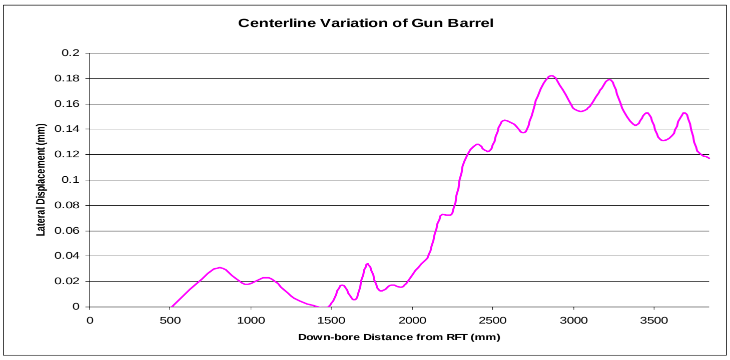


Figure 4 Centerline variation of a characteristic gun barrel

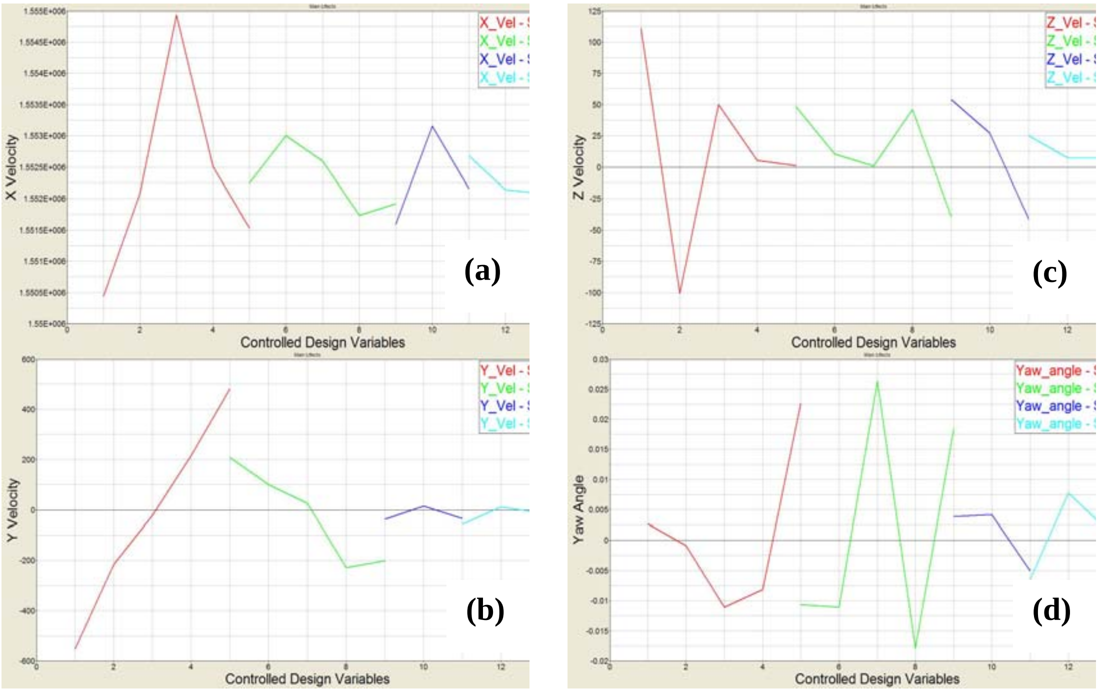
SENSITIVITY ANALYSIS

Three components, i.e., in axial (X), lateral (Y), and vertical (Z) directions, of rigid body velocity were obtained from each of the design cases. In order to compute yaw and pitch angles, a position vector for the tip of the projectile with respect to the center of gravity was derived, which represented the heading direction of the projectile at the exit. A summary of the statistical results is given in Table 2.

Table 2. A summary of response statistics based on 225 design cases

	X-Velocity (mm/sec)	Y-Velocity	Z-Velocity	Yaw Angle (degree)	Pitch Angle
Average	1.5523×10^6	-18.9	13.2	0.001	-0.00007
Std. Dev.	8.657×10^3	542.6	388	0.085	0.08
Maximum	1.5608×10^6	1449	2078	0.315	0.232
Minimum	1.4968×10^6	-3497	-1513	-0.263	-0.239

It can be seen that the contributions of gun barrel flexure to the deviations of X velocity was negligible because of a small coefficient of variation (0.5%) in the response. As expected, the mean values of the other responses were virtually zero since symmetric gun shapes, i.e. bent on both positive and negative sides, were utilized. Nevertheless, their standard deviations were all substantially high, implying strong sensitivity to the barrel centerline variations. To further understand how the muzzle responses reacted to a change in the levels of the design variables, the main effects [7] of each controlled factor was computed and given in Figure 5. The controlled design variables referred to the number of levels chosen for each barrel shape in DOE. Apparently, shape 1 demonstrated significant influences on X and Y velocities as opposed to the other two responses. The X



velocity reached a highest value of 1555 m/sec when the bore came with no deflection at the muzzle. Expectedly, the Y velocity was close to neutral for a perfectly straight bore and proportionally increased with the increase of bore variations as illustrated in Figure 5(b). However, the main effects of barrel shapes on Z velocity and yaw angle responses exhibited no obvious pattern.

Figure 5. Changes in (a) X-velocity, (b) Y-velocity, (c) Z-velocity, and (d) yaw angle by a change in the levels of the four barrel shapes.

In addition, analysis of variance (ANOVA) technique was also employed to estimate error variance and to determine the relative importance of various factors, i.e. barrel shapes and their interactions in this study. Figure 6 (a), (b), (c) and (d) shows the contributing percentage of barrel shapes to X (axial) velocity, Y (lateral) velocity, yaw angle, and pitch angle, respectively. A total of 11 contributing sources including error term were used in the bar charts. For X velocity, there was no outstanding winner, all shapes virtually having equal contributions. Regarding Y velocity, more than 50% of its variations could be explained by shape 1 alone, and the level of importance decreased with the location away from the muzzle. The cross terms among these four shapes shows very little effect on the response. Speaking of yaw angle, shape 4 surprisingly contributed more than 30% of its total variations while each of the other shapes did less than 20%. Similarly, from Figure 6 (d) the importance of shapes 1 and 4 appeared to be twice as much compared with that of the other two shapes to the pitch angle variations. The significance of shape 4 might be due to the fact that the peak acceleration took place at the location where the deflection was simulated, which led to higher lateral and vertical accelerations. As a result, it substantially affected the pitch and yaw angles at the muzzle.

A response surface model that fitted the observed 225 data was derived for each response. A complete high order polynomial that consists of a total of 64 regression coefficients, 4th order for shapes 1 and 2 due to five levels of data in use, and 2nd order for shapes 3 and 4 due to three levels of data, was adopted. The R-squared values for the fits of X velocity, Y velocity, yaw angle and pitch angle were 0.31, 0.64, 0.27 and 0.26, respectively. The values could be interpreted as the proportion in the data variability explained by the regression model. The results demonstrated that the Y velocity served the best candidate for prediction. Figure 7 (a), (b), (c) and (d) provides the residual plots for X velocity, Y velocity, yaw angle, and pitch angle, respectively. It depicts the differences between the observed response values from the exact analysis solver and the predicted response values from the regression model. A straight line shown in each plot represents the place where the predicted and observed values match each other. Overall speaking, the wide scatter of the residuals in the figures indicates that the response surface models failed to provide good predictions on the responses except Y velocity.

Finally, the interactions between design variables were also under investigation. An interaction plot between shapes 1 and 2 for Y velocity is given in Figure 8 (a). It explains the level of one factor to produce the same effect on the response at different levels of another factor. Because all five curves were not overlapped to one another, their interactions appeared to be weak. However, all levels of shape 1 dominated on the Y velocity, another evidence of shape 1 being the most significant factor. Figure 8 (b) gives the interactions between shapes 1 and 4 for yaw angle, which appeared to show strong correlations. A maximum yaw angle of 0.06 degree occurred when shape 1 at 0.25 mm level and shape 4 at zero deflection level. Nevertheless, the yaw angle turned to negative when shape 4 at -0.03 mm level. Equivalent effects can be estimated from the chart as well, for instance, shape 4 at 0.03 mm along with shape 1 at -0.25 mm or 0 mm level.

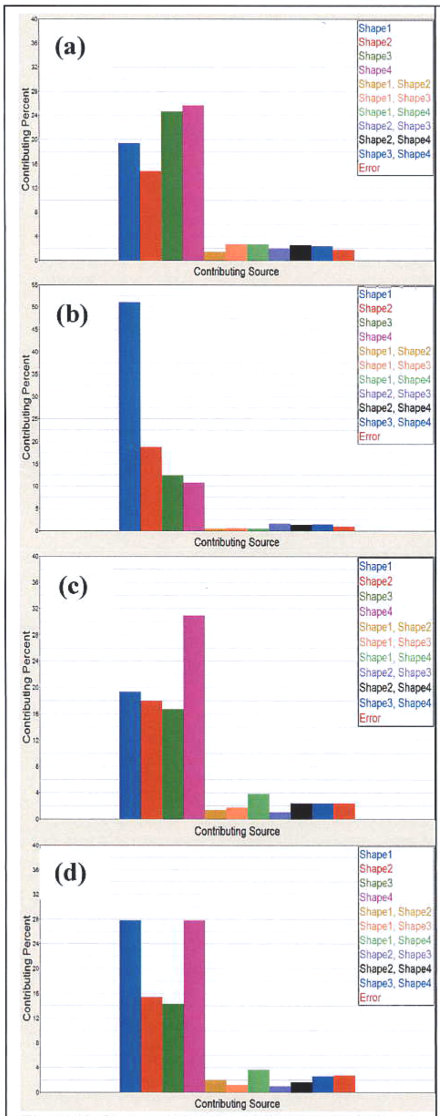


Figure 6. Comparison of contributing percentage of barrel shapes to (a) axial (X) velocity, (b) lateral (Y) velocity, (c) yaw angle, and (d) pitch angle.

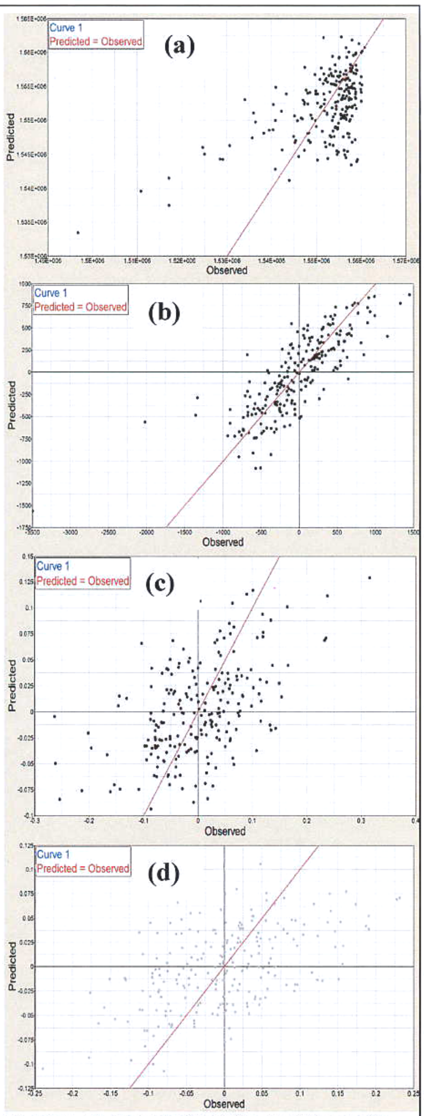


Figure 7. Residual plots of (a) axial (X) velocity, (b) lateral (Y) velocity, (c) yaw angle, and (d) pitch angle, respectively.

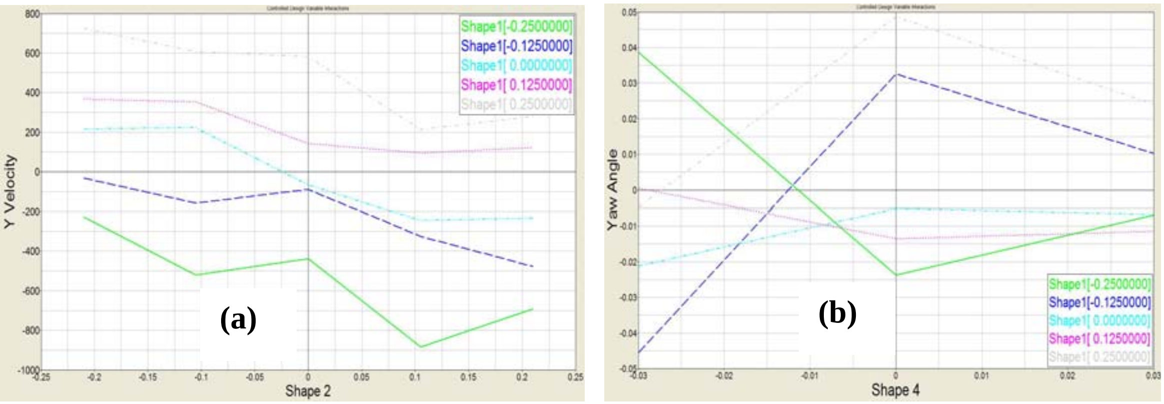


Figure 8. Interaction plots (a) between shapes 1 and 2 for Y velocity and (b) between shapes 1 and 4 for yaw angle, respectively.

SUMMARY

The centerline variations of a 60-mm gun barrel were modeled with four independent shape variables. The first half of the length on muzzle side that exhibited higher variations was represented by five levels of lateral displacements as much as 0.25 mm, while the other half typically having small deviations was characterized by three levels of deflections as much as 0.05 mm. DOE techniques were employed, which resulted in a study of 225 cases based on a full factorial design. The muzzle responses of three velocity components, and yaw and pitch angles were solved in each case. The computed results were then analyzed with ANOVA technique.

It was found that the exit lateral velocity was highly sensitive to shape 1, i.e. the deflection mode at the muzzle, while the other three shapes showed little contribution. The axial velocity was marginally affected by the barrel centerline variations, no outstanding distinction exhibited among the four shapes. A significant portion of exit yaw and pitch angles was found to be attributed by shape 4, i.e. the place where the projectile was accelerated the most, which resulted in substantial lateral and vertical accelerations.

REFERENCES

[1] C. Watson, J. D. Knapton, N. Boyer, I. and C. Stobie, Sensitivity of Muzzle Velocity Repeatability to Variations in Initial Conditions, *BRL-TR-3323*, Ballistic Research Laboratory, Aberdeen Proving Ground, MD, March 1992.

[2] J. Thomas, G. Wren, T. Coffee, and S. Richardson, Muzzle Velocity Sensitivity Based on Interior Ballistic Modeling, *ARL-MR-110*, US Army Research Laboratory, Aberdeen Proving Ground, MD, September 1993.

[3] W. Oberle and G. Wren, Estimating Muzzle Velocity Variability in a Regenerative Liquid Propellant Gun through Stochastic Modeling, *ARL-TR-746*, US Army Research Laboratory, Aberdeen Proving Ground, MD, June 1995.

[4] M. Bundy, J. Newill, V. Marcopoli, M Ng, and C. Wells, A methodology for characterizing gun barrel flexure due to vehicle motion, *Shock and Vibration*, Vol. 8, 223-228 (2001)

[5] T. Erline, Deviation Effect of an In-Bore Centerline on a 5-Inch Naval Gun, *ARL-MR-492*, US Army Research Laboratory, Aberdeen Proving Ground, MD, September 2000.

[6] M. Chen, “Structural Design and Analysis of Hit-to-Kill Projectile”, *9th International LS-DYNA Users Conference*, Dearborn, Michigan, USA, Session 2, 13-22 (2006)
[7] Help Manual of HyperStudy 7.0, Altair Engineering Inc. (2005)