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COMPUTATIONAL STUDY OF THE FLOW AROUND A PROJECTILE MOVING THROUGH A GUN BARREL

I. Mehmedagic[1], D. Carlucci[1], S. Thangam[2]

[1] *U.S. Armament, Research and Development Center, Picatinny Arsenal, Dover, NJ 07871, USA*
email: igbal.mehmedagic@pica.army.mil
[2] *Stevens Institue of Technology, Hoboken , 07828, USA, email:sthangam@stevens.edu*

Computation of the flow field around a projectile moving through a gun barrel is a complex problem. The complications are due to presence of numerous factors, such as turbulence, compressibility, anisotropy of the stress field, complexities due to surface geometry, and change of the computational domain in time. In this study, calculation of the flow characteristics around 155 mm moving projectile in gun barrel is undertaken. The flow is considered to be unsteady, compressible, turbulent, and nonisothermal. An anisotropic two-equation model for the Reynolds stress that was developed through a novel technique involving the representation of the energy spectrum and invariance based scaling is used in performing calculations. The model was implemented in a commercial code (Fluent) and its performance was validated with benchmark experimental flows. Computations are performed for stagnation pressures of up to 600 MPa, temperatures of up to 5000K, and free-stream velocities of about 1000 m/s in this study. The results are analyzed and the predictive capability of the model is discussed.

INTRODUCTION

Prediction of the flow field around a projectile moving through a gun barrel is very complex task. The complexities are due to presence of numerous factors, such as turbulence, anisotropy of the stress field, complex surface geometry, change in time of computational domain, and compressibility. Turbulent flow field past projectiles are known to exhibit a wide range of length and scales, and their accurate representation and modelling plays crucial role in the projectile's design. The current trend in simulation of turbulent flows for practical application relies heavily on computations based on Reynolds Averaged Navier-Stokes (RANS) equations. However, Reynolds stress tensor, embedded in these equations, has to be modelled. Most of these models are constructed using intuitive approximation or physical analogy rather than formal manipulation of Navier-Stokes equations [1], [2]. This approach often assumes turbulence to be characterized by a single length and time scale while in practice turbulence, at high Reynolds numbers, is exited by a wide range of time and length scales. In engineering practice the most commonly used Reynolds stress models are two-equation models due to their simplicity and robustness. However, to be of general validity, such models should incorporate the effects of system rotation, swirl, surface curvature, anisotropy and compressibility. Two equation models failed to predict rotating turbulence due to the break down of two basic assumptions implicit in these models- the Taylor approximation and the universal Kolmogorov spectrum [4]. By replacing the universal Kolmogorov energy spectrum with a rotation modified energy spectrum the eddy viscosity was reformulated. Based on the modified spectrum and the approach used by Reynolds [5], the equation for dissipation rate was also reformulated. An extended Kolmogorov phenomenology for compressible turbulence is used to obtain compressible constitutive relation for the Reynolds stresses. The interaction of vortical and acoustic modes affects the solenoidal component of the energy spectrum which characterizes the vortical turbulence and is responsible for the Kolmogorov energy cascade. An eddy viscosity and a constitutive relation for compressible Reynolds stress are derived using this compressibility corrected solenoidal energy spectrum. A consistent anisotropic two-equation model for the Reynolds-stress is developed with the aid of invariance principles and available scaling laws and used in the present investigation [6].

MODEL FORMULATION

A general constitutive relation for Reynolds stress τ_{ij} is considered in the following form:

$$\boldsymbol{b}_{ij} = \left(\boldsymbol{\tau}_{ij} - \frac{2}{3}\boldsymbol{K}\boldsymbol{\delta}_{ij}\right)/2\boldsymbol{K} = f(\boldsymbol{S}_{ij}^*, \boldsymbol{W}_{ij}^*) \tag{1}$$

where

$$\boldsymbol{S}_{ij}^* = \left(\frac{\boldsymbol{K}}{\boldsymbol{\varepsilon}}\right)\boldsymbol{S}_{ij}; \boldsymbol{S}_{ij} = \frac{1}{2}\left(\partial_j \boldsymbol{u}_i + \partial_i \boldsymbol{u}_j\right)$$

$$\boldsymbol{W}_{ij}^* = \left(\frac{\boldsymbol{K}}{\boldsymbol{\varepsilon}}\right)\boldsymbol{W}_{ij}; \boldsymbol{W}_{ij} = \frac{1}{2}\left(\partial_j \boldsymbol{u}_i - \partial_i \boldsymbol{u}_j\right)$$

are normalized rate of strain tensors.

A general form of polynomial representation for $\mathbf{b}_{ij}$ [7]is given as:

$$\boldsymbol{b} = \sum_{\lambda} \boldsymbol{G}^{\lambda} \boldsymbol{T}^{\lambda} \tag{2}$$

Where, $\boldsymbol{T}^{\lambda}$ are the integrity bases,

$$\boldsymbol{T}^{(1)} = \boldsymbol{S}^{*}; \boldsymbol{T}^{(2)} = \boldsymbol{S}^{*}\boldsymbol{W}^{*} - \boldsymbol{W}^{*}\boldsymbol{S}^{*}$$

$$\boldsymbol{T}^{(3)} = \boldsymbol{S}^{*2} - \frac{1}{3}\left\{\boldsymbol{S}^{*2}\right\}\boldsymbol{I}; \boldsymbol{T}^{(4)} = \boldsymbol{W}^{*2} - \frac{1}{3}\left\{\boldsymbol{W}^{*2}\right\}\boldsymbol{I}$$

$$\boldsymbol{T}^{(5)} = \boldsymbol{W}^{*}\boldsymbol{S}^{*2} - \boldsymbol{S}^{*}\boldsymbol{W}^{*2}; \boldsymbol{T}^{(6)} = \boldsymbol{W}^{*2}\boldsymbol{S}^{*} + \boldsymbol{S}^{*}\boldsymbol{W}^{*2} - \frac{2}{3}\left\{\boldsymbol{S}^{*}\boldsymbol{W}^{*2}\right\}\boldsymbol{I}$$

$$\boldsymbol{T}^{(7)} = \boldsymbol{W}^{*}\boldsymbol{S}^{*}\boldsymbol{W}^{*2} - \boldsymbol{W}^{*2}\boldsymbol{S}^{*}\boldsymbol{W}^{*}; \boldsymbol{T}^{(8)} = \boldsymbol{S}^{*}\boldsymbol{W}^{*}\boldsymbol{S}^{*2} - \boldsymbol{S}^{*2}\boldsymbol{W}^{*}\boldsymbol{S}^{*}$$

$$\boldsymbol{T}^{(9)} = \boldsymbol{W}^{*2}\boldsymbol{S}^{*2} + \boldsymbol{S}^{*2}\boldsymbol{W}^{*2} - \frac{2}{3}\left\{\boldsymbol{S}^{*2}\boldsymbol{W}^{*2}\right\}\boldsymbol{I}; \boldsymbol{T}^{(10)} = \boldsymbol{W}^{*}\boldsymbol{S}^{*2}\boldsymbol{W}^{*2} - \boldsymbol{W}^{*2}\boldsymbol{S}^{*2}\boldsymbol{W}^{*}$$

Where, $\{\ldots\}$ indicates the trace, and $\mathbf{I} = \boldsymbol{\delta}_{\mathbf{ij}}$.
In explicit algebraic stress models [8], the unknown scalar functions, $\boldsymbol{G}^{\lambda}$ are obtained based on second order closure theories. The basic form of the scalar functions, $\boldsymbol{G}^{\lambda}$, may also determined by the use of scaling laws. This procedure is employed using a representation of eddy viscosity that includes the effect of rotation [4]:

$$\boldsymbol{v}_T = \boldsymbol{C}_6 \frac{\boldsymbol{K}^2}{\boldsymbol{\varepsilon}} / \sqrt{1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*}} \tag{3}$$

$$\boldsymbol{W}_{ij}^{*} = \left(\frac{\boldsymbol{K}}{\boldsymbol{\varepsilon}}\right)\left(\boldsymbol{W}_{ij} + \alpha\varepsilon_{mji}\boldsymbol{\Omega}_m\right); \boldsymbol{W}_{ij} = \frac{1}{2}\left(\partial_j \boldsymbol{u}_i - \partial_i \boldsymbol{u}_j\right)$$

Here, $\boldsymbol{\Omega}_{\mathrm{m}}$ is the angular velocity of the noninertial frame relative to an inertial frame, $\boldsymbol{\alpha} \approx$ *2.25*; and the coefficients $C_6 \approx 0.1$, $C_7 \approx 0.2$, [4]. The corresponding length scale, $\boldsymbol{l}^{*}$ is

$$\boldsymbol{l}^{*} = \boldsymbol{C}\frac{\boldsymbol{K}^{\frac{3}{2}}}{\boldsymbol{\varepsilon}} / \sqrt{1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*}}$$

If length scales in the linear term and higher order terms in the model are dimensionally consistent, it follows that the resulting expressions for the scalar functions, $\boldsymbol{G}^{\lambda}$ can be written as:

$$\begin{aligned}
&\boldsymbol{G}^{(1)} \propto \frac{\boldsymbol{K}^2}{\boldsymbol{\varepsilon}} / \sqrt{1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*}}; \boldsymbol{G}^{(2)}, \boldsymbol{G}^{(3)}, \boldsymbol{G}^{(4)} \propto \frac{\boldsymbol{K}^3}{\boldsymbol{\varepsilon}^2} / (1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*}) \\
&\boldsymbol{G}^{(5)}, \boldsymbol{G}^{(6)} \propto \frac{\boldsymbol{K}^4}{\boldsymbol{\varepsilon}^3} / (1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*})^{\frac{3}{2}}; \boldsymbol{G}^{(7)}, \boldsymbol{G}^{(8)}, \boldsymbol{G}^{(9)} \propto \frac{\boldsymbol{K}^5}{\boldsymbol{\varepsilon}^4} / (1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*})^2 \\
&\boldsymbol{G}^{(10)} \propto \frac{\boldsymbol{K}^6}{\boldsymbol{\varepsilon}^5} / (1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^{*}\boldsymbol{W}_{ij}^{*})^{\frac{5}{2}}
\end{aligned} \tag{4}$$

The general form of constitutive relations for three-dimensional flows is restricted to third order, the lowest order for characterizing three-dimensional features.
The Reynolds stress, $\boldsymbol{\tau}_{ij}$ can be written as:

$$\boldsymbol{\tau}_{ij} = \frac{2}{3}\boldsymbol{K}\boldsymbol{\delta}_{ij} - \frac{2\boldsymbol{C}^{(1)}\frac{\boldsymbol{K}^2}{\boldsymbol{\varepsilon}}}{\sqrt{1+\boldsymbol{C}_7\boldsymbol{W}_{ij}^*\boldsymbol{W}_{ij}^*}}\boldsymbol{S}_{ij} + \frac{\boldsymbol{C}^{(2)}\frac{\boldsymbol{K}^3}{\boldsymbol{\varepsilon}^2}}{\left(1+\boldsymbol{C}_7\boldsymbol{W}_{ij}^*\boldsymbol{W}_{ij}^*\right)}\left(\boldsymbol{S}_{ik}\boldsymbol{W}_{kj} - \boldsymbol{W}_{ik}\boldsymbol{S}_{kj}\right)$$
$$+ \frac{\boldsymbol{C}^{(3)}\frac{\boldsymbol{K}^3}{\boldsymbol{\varepsilon}^2}}{\left(1+\boldsymbol{C}_7\boldsymbol{W}_{ij}^*\boldsymbol{W}_{ij}^*\right)}\left(\boldsymbol{S}_{ik}\boldsymbol{S}_{kj} - \tfrac{1}{3}\boldsymbol{S}_{km}\boldsymbol{S}_{km}\delta_{ij}\right) + \frac{\boldsymbol{C}^{(4)}\frac{\boldsymbol{K}^3}{\boldsymbol{\varepsilon}^2}}{\left(1+\boldsymbol{C}_7\boldsymbol{W}_{ij}^*\boldsymbol{W}_{ij}^*\right)}\left(\boldsymbol{W}_{ik}\boldsymbol{W}_{kj} - \tfrac{1}{3}\boldsymbol{W}_{km}\boldsymbol{W}_{km}\boldsymbol{\delta}_{ij}\right)$$
$$+ \frac{\boldsymbol{C}^{(5)}\frac{\boldsymbol{K}^4}{\boldsymbol{\varepsilon}^3}}{\left(1+\boldsymbol{C}_7\boldsymbol{W}_{ij}^*\boldsymbol{W}_{ij}^*\right)^{\frac{3}{2}}}\left(\boldsymbol{W}_{ik}\boldsymbol{S}_{kl}\boldsymbol{S}_{lj} - \boldsymbol{S}_{ik}\boldsymbol{S}_{kl}\boldsymbol{W}_{lj}\right) \tag{5}$$
$$+ \frac{\boldsymbol{C}^{(6)}\frac{\boldsymbol{K}^4}{\boldsymbol{\varepsilon}^3}}{\left(1+\boldsymbol{C}_7\boldsymbol{W}_{ij}^*\boldsymbol{W}_{ij}^*\right)^{\frac{3}{2}}}\left(\boldsymbol{W}_{ik}\boldsymbol{W}_{kl}\boldsymbol{S}_{lj} - \boldsymbol{S}_{ik}\boldsymbol{W}_{kl}\boldsymbol{W}_{lj} - \frac{2}{3}\boldsymbol{S}_{ik}\boldsymbol{W}_{kl}\boldsymbol{W}_{li}\boldsymbol{\delta}_{ij}\right)$$

Coefficient calibration using data of [10] gave $C^{(1)} \approx 0.23$, $C^{(2)} \approx 0.01$, $C^{(4)} \approx 1.2$, $C^{(6)} \approx$ -0.06,and based on axisymmetric expansion and fully-developed pipe flow, gave: $C^{(3)} \approx$ 0.1, $C^{(5)} \approx 0.1$.Transport equation for $\boldsymbol{K}$ and $\boldsymbol{\varepsilon}$ are modeled in the following form

$$\left(\partial_t + \boldsymbol{U}_j\partial_j\right)\boldsymbol{K} = -\tau_{ij}\partial_j\boldsymbol{U}_i - \boldsymbol{\varepsilon} + \partial_i\left[\left(\boldsymbol{\alpha}_k\boldsymbol{v}_T + \boldsymbol{v}\right)\partial_i\boldsymbol{K}\right] \tag{6}$$

$$\left(\partial_t + \boldsymbol{U}_j\partial_j\right)\boldsymbol{\varepsilon} = -C_{\varepsilon 1}^*\frac{\boldsymbol{\varepsilon}}{\boldsymbol{K}}\tau_{ij}\partial_j\boldsymbol{U}_i - C_{\varepsilon 2}^*\frac{\boldsymbol{\varepsilon}^2}{\boldsymbol{K}} + \partial_i\left[\left(\boldsymbol{\alpha}_\varepsilon\boldsymbol{v}_T + \boldsymbol{v}\right)\partial_i\boldsymbol{\varepsilon}\right] \tag{7}$$

Where the coefficients,

$$C_{\varepsilon 1}^* = 1.42 - \frac{\eta\left(1-\dfrac{\eta}{\eta_0}\right)}{1+\beta\eta^2}\left(1+0.07\frac{\boldsymbol{P}}{\boldsymbol{\varepsilon}}\right); C_{\varepsilon 2}^* = \frac{\left(1.68C_3 + \dfrac{8}{3}\dfrac{\boldsymbol{K}\Omega}{\boldsymbol{\varepsilon}}\right)}{C_3 + \dfrac{\boldsymbol{K}\Omega}{\boldsymbol{\varepsilon}}}$$

with $\boldsymbol{\eta}_0$=5.6, $\boldsymbol{\beta}$ =0.138, $C_3 \approx 1.53$; $\boldsymbol{\eta}=(\boldsymbol{K}/\boldsymbol{\varepsilon})\sqrt{(2\boldsymbol{S}_{ij}\boldsymbol{S}_{ij})}$; $\boldsymbol{P}= -\boldsymbol{\tau}_{ij}\boldsymbol{S}_{ij}$ and $\boldsymbol{\alpha}_K \approx \boldsymbol{\alpha}_\varepsilon \approx 1.39$, are derived on a double-expansion procedure and RNG to account for anisotropic production that include the effects of rotation [9], [4]. Similar approach can be used to obtain the anisotropic Reynolds stress tensor and the eddy viscosity [11], and it is expressed as

$$\boldsymbol{v}_T = \boldsymbol{C}_6 \frac{\boldsymbol{K}^2}{\varepsilon} / \sqrt{1 + \boldsymbol{C}_7 \boldsymbol{W}_{ij}^* \boldsymbol{W}_{ij}^*} \left(1 + A^* M_t^2\right) \tag{8}$$

Where M_t is turbulent Mach number, and A^* is modeling coefficient determined by fixed point analysis of compressible homogenous turbulence. Using, again, the general polynomial representation of constitutive relation for Reynolds-stress (2) we obtained corrected $\boldsymbol{\tau}_{ij}$ for compressible flow in similar form as (5) with correction by $(1 + A^* M_t^2)$, i=1, 2, 3. The model developed in this way is of general, tensorially invariant form based on scaling laws and accounts for the effects of rotation, swirl, surface curvature, and compressibility. It had been earlier validated against experimental results for axially rotating pipe, rotating channel, mixing layer, etc.

SIMULATION AND RESULTS

In this study, calculation of the flow characteristics around 155 mm moving projectile with blow-by in gun barrel is undertaken. The flow is considered to be unsteady, compressible, turbulent, and nonisothermal. Simulations were performed using a commercially available Finite Volume Code (Fluent), with the above formulated model incorporated into the code via user define function (UDF). Figure 1. presents geometry and meshing of the model (using the Fluent pre-processor Gambit).

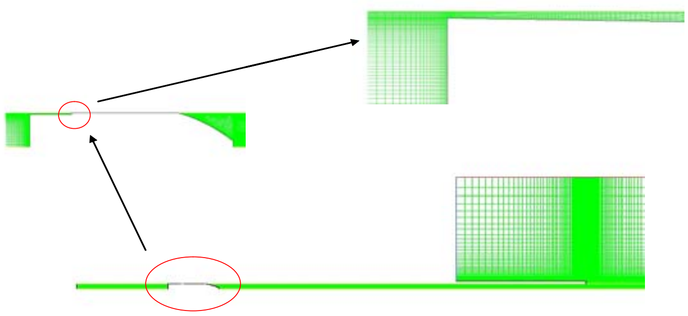

Figure 1. Meshed Model of 155 mm projectile with blow-buy in the gun barrel

As the projectile moves down the barrel formation of the shock wave is expected to occur once projectile accelerated enough. Figure 2 presents the Mach number contour

Figure 2. Oblique shock around nose of the projectile

around nose of the projectile. The turbulence model, as it can be seen in above figure, was capable of capturing the oblique shock, as well as reflected waves from the gun barrel. The pressure contour at the projectile base is presented by Figure 3. As the projectile progressed from starting position where initial pressure was 4287 atm., the pressure at base of the projectile regressed due to the movement of the projectile. Blow-by section of this set up at the narrowest place has the width of the only 125 microns. As the projectile starts moving products of combustions expand and rush into this space where jump in the velocity occurs and flow turns supersonic. This section diverges as it moves toward the nose and as expected from compressible gas theory it accelerates which is clearly visible in the Figure 4.

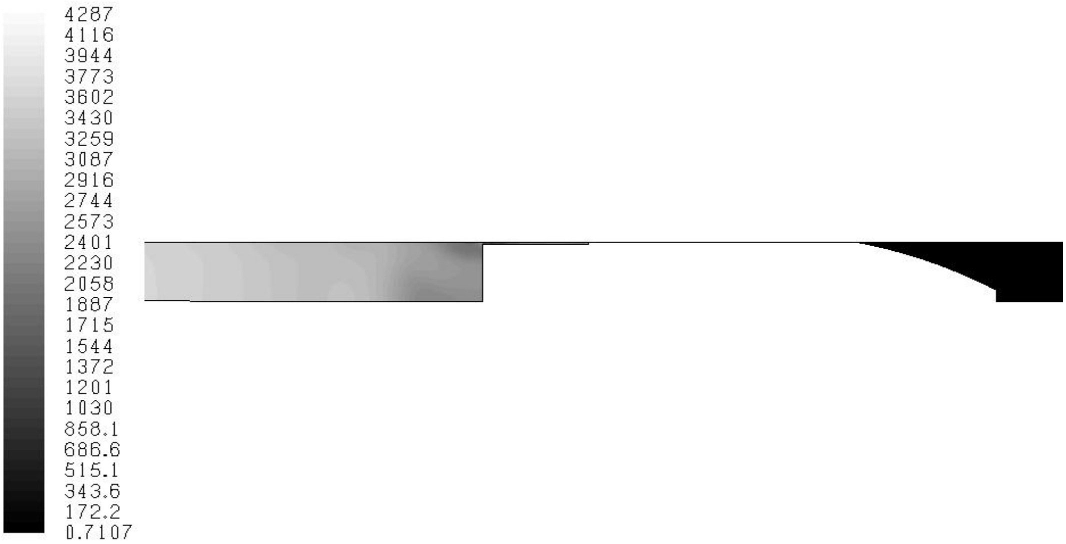

Figure 3. Pressure Distribution behind the Projectile

Figure 4. Mach Number Contour for the Blow-by

CONLUSIONS AND FUTURE WORK

Anisotropic Reynolds stress model developed using a phenomenological approach was implemented into the commercial code Fluent and used for the simulation of 155 mm moving projectile with blow-by down the gun barrel. The model preserves the two-equation form and meets the invariance constraints and includes the rotation-modified energy spectrum and a generalized constitutive representation that does not require the use of local equilibrium assumption. Model coefficients were obtained with fixed-point analysis of simple flows and using empirical data. Previous validation has shown that model is capable of capturing the effects of rotation, curvature and compressibility. In this paper the model was used to predict the dominant features of the flow field around the moving projectile. Future work will focus on the use of finer mesh to accurately capture the turbulence parameters. A detailed comparison with experimental data is also planned for further validation of the model coefficients.

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