DESIGN OPTIMIZATION OF SOLID PROPELLANT ROCKET MOTOR

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This work is focused on the method of the design optimization of a solid propellant rocket motor with respect to the rocket arrangement and the main required rocket flight parameters, i.e. the maximal firing range or final rocket velocity at the end of rocket motor operation. The rocket motor design solution can be divided into two parts. The first one solves the relations among maximum firing range or total rocket velocity, the total impulse to the initial total mass ratio, ballistic coefficient, rocket motor specific impulse and propellant mass to the final mass ratio. These relations cannot be determined directly but can be effectively rated by the numerical solution of the main task of exterior ballistics for chosen rocket parameters and type of trajectory. In the second part of the SPRM design the above parameters influence the calculation of combustion chamber filling coefficient, which depends on individual solid propellant charge shape. Principle example of derivation is carried out for the star shape of the grain.

INTRODUCTION

The aim of this paper is to present a method of design that serves for a more accurate determination of the ballistic, weight and geometric characteristics of the solid propellant rocket motor (SPRM) for the given tactical and technical requirements. Such method can be effectively used for the design of SPRM being a propulsion unit of the transportation of given effective payload (i.e. rocket warhead) to the target in the determined maximum range, or which will accelerate the given payload to the required final velocity.

At the beginning of the solution, it is necessary to accept the rocket conception. Tactical technical requirements have to contain, besides the mentioned principle, data as manner of the rocket flight to the target (i.e. either guided or unguided), demands laid on rocket acceleration, convenient construction materials for individual parts, etc. Based on these requirements, the rocket shape, total rocket arrangement and character of individual parts can be chosen. The aim of this task is to estimate the necessary total impulse of SPRM, the total rocket weight and the weight of solid propellant (SP) that are necessary for rating the SPRM dimensions and ballistic parameters.

The principle of SPRM design can be explained by the general scheme of the ballistic design, presented in **Fig. 1** [1]. The solution of the rocket motor design can be divided into two parts.

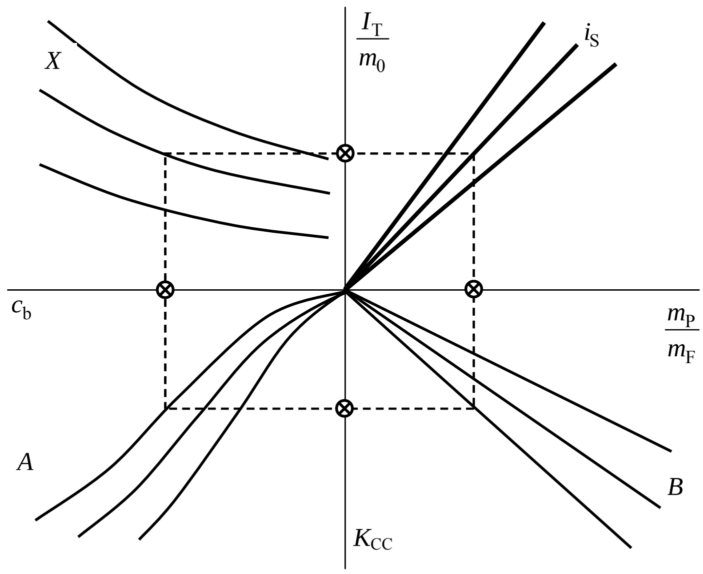


Fig. 1. General scheme for ballistic design solution

The first one solves the relations among the maximum fire range *X*, the total impulse to initial total mass ratio $\frac{I_T}{m_0}$, the ballistic coefficient c_b, the specific impulse i_S and the propellant mass to final mass ratio $\frac{m_P}{m_F}$. These relations cannot be determined directly, but can effectively be rated by the main task of the exterior ballistics solution for given rocket parameters and the type of trajectory. The principal solution is outlined in the upper half part of the graph on **Fig. 1**.

In the second part of the SPRM design the parameters as $\frac{I_T}{m_0}, c_b, i_S, \frac{m_P}{m_F}$ influence the calculation of the combustion chamber filling coefficient through the functions *A* and *B* that depend on the individual solid propellant charge shape.

THE ROCKET TRAJECTORY SOLUTION

For the rocket trajectory solution, the normal atmospheric conditions are assumed. Further let us presuppose that the rocket moves in the plane of firing, i.e. it passes in vertical plane through the points of the rocket start and the target. The rocket is considered unguided. The solution in such case will be more convenient for the case assumed.

The input data for the solution of ballistic rocket trajectory are – rocket caliber – *D*, initial angle of longitudinal inclination, valid for the maximum range $-\theta_{0\,max}$, rocket shape coefficient valid for the respective air resistance law – *i*, specific impulse – i_S, initial rocket weight – m_0 and total impulse of the RM $-I_T$.

In this phase of solution, RM thrust can be estimated as a function of the initial rocket weight and the chosen rocket acceleration at the point of rocket start $-a_0$. The initial acceleration for the existing powerful rockets moves within the range $a = (250 \div 350)\ ms^{-2}$. The time of RM operation is then determined from the RM total impulse divided by rocket motor thrust.

The main task of the external ballistics is solved by the numerical integration of the set of equations, describing the rocket trajectory in either a Cartesian or polar co-ordinate system. The ballistic design of the rocket, [1] results from the determination of the maximum firing range or from the final rocket velocity as function of the total SPRM impulse I_T and the initial rocket mass (weight) m_0. Firing range or final rocket velocity can be determined by numerical solution of known differential equations system describing the unguided rocket trajectory [1].

$$F = a_0 m_0 ; \tag{1}$$

$$t_k = \frac{I_T}{F} ; \tag{2}$$

$$\frac{dv}{dt} = \frac{F}{m} - c_b\, H_\tau(y)\, v\, G(v_\tau) - g \sin\theta ; \tag{3}$$

$$\frac{d\theta}{dt} = -g \frac{\cos\theta}{v} ; \tag{4}$$

$$\frac{dx}{dt} = v \cos\theta ; \quad \frac{dy}{dt} = v \sin\theta ; \tag{5}$$

$$\frac{dm}{dt} = -\frac{F}{i_s} . \tag{6}$$

The solution is of need to choose the initial rocket acceleration a_0 and the SPRM specific impulse i_S. The initial angle of elevation θ_0 has to correspond either to the maximum firing range or to the principal requirements for the rocket trajectory at the required final rocket velocity. The further proposal for the booster rocket motor for anti-tank or air-defense guided rocket will be assumed when reaching the required final velocity for a chosen angle of inclination is necessary. The ratio of SP charge mass and final rocket mass will be:

$$\frac{m_p}{m_F} = \frac{\frac{I_T}{m_0}}{i_S - \frac{I_T}{m_0}} .. \tag{7}$$

This ratio of masses is very important for the further – internal ballistics part of solution.

PRINCIPLE OF THE ROCKET MOTOR DESIGN

The dimensions of the SP charge can be calculated as function of the combustion chamber filling coefficient [2]:

$$K_{CC} = \frac{A_{SP}}{A_{CC}}, \tag{8}$$

where A_{SP} is the solid propellant charge cross section area nearest the nozzle, A_{CC} is the inner cross section area of the empty combustion chamber. From the structure point of view the ratio of SP charge weight and final rocket mass is follows [3]:

$$\frac{m_P}{m_F} = \frac{aK_L K_{CC}}{C_N + bK_L}, \tag{9}$$

where K_L is SP charge slenderness ratio, a and b are factors taking into account SP density and densities and strength factors of the combustion chamber structure, C_N is relative invariable mass [3].

The next solution has to be derived from the condition for the individual SP charge shape. For example the star grain charge shape is chosen in accordance to **Fig. 2**.

The slenderness ratio for this SP charge shape is given in following form [4]:

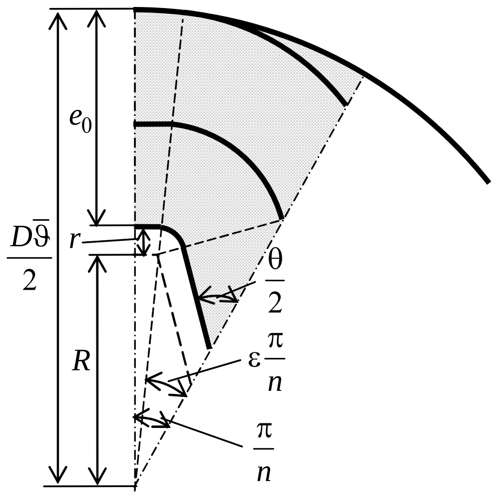
$$K_L = K_S\left(1 - K_{CC}\right), \tag{10}$$

where K_S is the shape coefficient.

The usually used solution in the research works is based on the maximum attainable velocity, which can be found as the extreme function as follows:

$$\frac{dv_F}{dK_{CC}} = \frac{d}{dK_{CC}}\left(\frac{m_P}{m_F}\right) = 0\,. \tag{11}$$

After applying the necessary mathematical procedures, the basic relation for relative invariable mass can be obtained [4]



Fi. 2. The scheme of the SP charge having the channel in the shape of n-tips star

$$C_N = \frac{m_N}{D} = bK_S\,\frac{(1-K_{CC})^2}{(2K_{CC}-1)}\,. \tag{12}$$

Equation (3) can be then transformed as follows:

$$\frac{m_P}{m_F} = \frac{a}{b}(2K_{CC}-1) \tag{13}$$

and the needed combustion chamber filling coefficient will be given by the eqaution

$$K_{CC} = 0.5\left(\frac{b}{a}\frac{m_P}{m_F} + 1\right). \tag{14}$$

The further solution flows out from the ballistic coefficient definition [4], which is a function of the rocket calibre, coefficient of rocket shape *i* and final rocket mass.

$$c_b = \frac{1000\, i\, D}{m_F}. \tag{15}$$

From It can be seen from equations (6) and (9) after some arrangement. Then we shall get the relation for invariable mass as follows:

$$m_N = \frac{m_F\left(1 - K_{CC}\right)}{K_{CC}}, \tag{16}$$

which closes this step of solution.

SOLUTION PROCEDURE

The rocket with SPRM proposal flows out from the allocation, which contains the required maximum rocket firing range or final velocity and the required invariable mass which takes into account the mass of payload (rocket warhead) and masses of rocket parts, that don't depend on the RM length. Parameters that are required for solution: the rocket caliber, SPRM specific impulse and required initial rocket acceleration. Further, the set of SPRM total impulse values and independent set of initial rocket mass values is chosen.

We can proceed in the following sequence. For each pair of values $\left(I_C, m_0\right)$ by numerical solution of the external ballistic main task the maximum rocket firing range or final velocity is determined. The result is the matrix of reached values of maximum rocket firing ranges or final velocities. As example we can carry out the calculation of rocket with D = 0.15 m and required firing range X = 35 km. The calculated values of firing range are introduced in table 1.

Tab. 1

	m_0				
I_T	80.0	90.0	100.0	110.0	120.0
80000	38030	33989	30713	28044	25866
82500	41067	36313	32563	29571	27136
85000	44442	38943	34627	31225	28512
87500	48262	41856	36895	33062	30028
90000	52444	45109	39454	35081	31653

Further on, we assign from this matrix by interpolation in matrix rows to each value of total impulse the corresponding initial rocket mass for required maximum rocket firing range or final velocity. For performance of interpolation in every matrix row the condition $\left(\varphi_{i,j}\right)_{MIN} \leq \varphi_{req} \leq \left(\varphi_{i,j}\right)_{MAX}$ has to be met. If such a demand isn't fulfilled, it is necessary to go back to the solution beginning and to select the other extents of values I_T or m_0.

For final set of values I_T and m_0 pairs we will get the corresponding values m_{Pi}, m_{Fi} and (m_{Pi}/m_{Fi}), the values of total combustion chamber filling coefficients K_{CCi} and the values of the rocket invariable mass m_{Ni}. The resulting values of solution are shown in table 2.

Tab. 2

I_T Ns	m_0 kg	m_P kg	m_F kg	K_{CC}	m_N kg
80000	87.28	38.09	49.19	0.6101	31.44
82500	93.24	39.29	53.95	0.6035	35.45
85000	99.04	40.48	48.52	0.5982	39.34
87500	104.43	41.67	62.76	0.5940	43.03
90000	110.21	42.86	67.55	0.5904	46.72

By interpolation for required invariable mass can be determined the most convenient values of the SPRM total impulse, as well as the rocket initial mass. With the help of these values the most convenient value of combustion chamber filling coefficient K_{CC} is determined, being after that applied for the SP charge final dimensions calculation. The resulting values for chosen required value of the invariable rocket mass $m_N = 40\ kg$ are as follows: I_T = 85455 Ns; m_0 = 100.05 kg; m_P = 40.69 kg; m_F = 59.36; K_{CC} = 0.5974.

When checking the accuracy of the design in this example, we will get the resulting value of maximum firing range X – 34548 m. It is very good result of this design step.

CONCLUSION

It is obvious that the SPRM design carried out by the above introduced method it is the greater accuracy due to the use more accurate method of the fire range determination. The maximum fire range obtained from checking calculation of the trajectory after termination of the design is very near to the required firing range.

The rocket motor ballistic design is considered as preliminary design. Therefore the rocket and rocket motor design can be gradually improved in the consequential steps of the rocket project.

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