# EXPERIMENTAL AND NUMERICAL STUDY OF THE SPIN OF THE HARDCORE IN A PROJECTILE DURING ACCELERATION IN A GUN BARREL 

Eva K. Friis ${ }^{1}$, Øyvind Frøyland ${ }^{2}$ and John F. Moxnes ${ }^{3}$<br>1 Nammo Raufoss AS, P.O. Box 162, N-2831 Raufoss, NORWAY, Phone: +47<br>61153609, E-mail: eva.friis@nammo.com<br>2\&3 FFI (Norwegian Defence Research Establishment), P.O. Box 25, N-2007 Kjeller, NORWAY, Phone: +47 63807514, E-mail: John-F.Moxnes@ffi.no e


#### Abstract

A new feasibility of modern aircrafts is to attack targets on the ground at long ranges. In this paper we study the use of a Tungsten Carbide penetrator in full bore ammunition. The Tungsten Carbide penetrator, also referred to as the hardcore, has superb penetration capabilities in armor steel. It is important that the hardcore spins up with the same rate as the rest of the projectile, if not the projectile could become unstable during flight. In this paper we study a design solution which utilizes friction to spin up the hardcore. During acceleration in the gun barrel setback forces will press the hardcore into the steel base of the projectile. If the coefficient of friction between the hardcore and the steel is large enough, the shear stress will cause the hardcore to spin up together with the projectile. The results suggest that both the configuration of the hardcore and the friction coefficient are important parameters.


## INTRODUCTION

A new feasibility of modern aircrafts is to attack targets on the ground at long ranges. We study the use of a Tungsten Carbide penetrator in full bore ammunition for penetration in armored targets. If the penetrator, also referred to as the hardcore, does not spin up with the same rate as the projectile during launch, the projectile could become unstable during flight.

In this article we study a design solution which utilizes friction to spin up the hardcore. During acceleration in the gun barrel setback forces will press the hardcore into the steel base of the projectile. If the coefficient of friction between the hardcore and the steel is large enough, the shear stress will cause the hardcore to spin up together with the projectile.

To achieve sufficient shear stresses during spin up, the contact area, the rifling twist, the coefficient of friction and the acceleration is important to control.

A short outline of the paper: First we establish some analytical relations related to the necessary torque to spin up the projectile. Thereafter we establish some analytical formulas related to the rifling pitch. Then we show how the measured breech pressure during a shot is used to find the base pressure on the projectile. Finally we show simulation results.

## ANALYTICAL CALCULATIONS FOR DIFFERENT HARDCORES

Figure 1 shows a sketch of hard core with flat base.


Figure 1: The hardcore with flat base. The hardcore is resting on a base of steel inside the projectile

Consider the situation where a hardcore of calibre $2 R$ is accelerated in the axial direction due to the steel base. We consider the hardcore to be approximately a cylinder, and also assume that only the contact surface at the base is of importance for acceleration and spinning up the projectile. The normal stress $\sigma_{n}$ on the contact area with normal vector in the axial direction of the cylinder is equal to the axial stress $\sigma_{a}$ and becomes

$$
\begin{equation*}
\sigma_{n}=\sigma_{a}=m a / \pi R^{2} \tag{1}
\end{equation*}
$$

Where $m$ is the mass of the cylinder, $R$ is the radius of the cylinder and $a$ is the acceleration of the cylinder along the axial direction (out of the gun). The shear force during the twist due to spin is equal to $\sigma_{\mathrm{s}} \stackrel{\text { def }}{=} \mu \sigma_{n}$ where $\mu$ is the coefficient of friction. The equation for the torque then gives

$$
\begin{equation*}
T \stackrel{d e f}{=} \int_{S} \sigma_{s} x r d s=I \dot{\omega} \tag{2}
\end{equation*}
$$

where $r$ is the radial distance from the axis of the cylinder. $T$ is the torque. The integration is over the contact surface of the cylinder and the steel base. $I$ is the moments of inertia and is given by $I=(1 / 2) m R^{2}$ for a cylinder. $\omega$ is the angular acceleration.

Assuming a constant normal stress (although varying with time) and a constant coefficient of friction gives when applying the integration in eq. (2) over the contact area and using eq. (1) that

$$
\begin{equation*}
T=\int_{S} \mu \frac{m a}{\pi R^{2}} \times r d S=\frac{2 \mu m a R}{3}=\frac{1}{2} m R^{2} \dot{\omega} \Rightarrow \dot{\omega}=\dot{\omega}_{\max } \stackrel{\operatorname{def}}{=} \frac{4 \mu a}{3 R} \tag{3}
\end{equation*}
$$

$\dot{\omega} \max$ is the maximum angular acceleration the cylinder can achieve for a given coefficient of friction, axial acceleration and radius R. If during the axial acceleration in the gun barrel the angular acceleration of the projectile is larger than this value, the spin of the cylinder (hardcore) will not follow the spin of the projectile and the projectile will become unstable during flight. Thus the criterion for normal functioning of the projectile is that the angular acceleration of the projectile fulfills the threshold condition

$$
\begin{equation*}
\dot{\omega} \leq \dot{\omega}_{\max }=\frac{4 \mu a}{3 R} \tag{4}
\end{equation*}
$$

This relation must be fulfilled during the whole axial acceleration phase in the gun barrel. Notice from the formula that the mass of the hardcore is absent. The reason for this is that although the normal force becomes higher for larger masses due to inertia, the moments of inertia also become higher. A full cancellation is achieved. The radial dimension $R$ is taking part in the formulae. A larger cylinder gives a smaller threshold, i.e. if the caliber increases only, the hardcore could more easily slip the steel base leading to malfunction. We finally observe that the threshold in eq. (4) is directly proportional to the coefficient of friction.

Consider now a hardcore with a conical base, as shown in figure 2. The projectile design is such that the conical surface is the only contact area with the steel. The half angle of the cone is called $\alpha$. Assuming axial stresses due to the axial acceleration gives that the normal stress on the surface of the cone becomes $\sigma_{a} \operatorname{Sin}(\alpha)$. But the hardcore is a material that also gives radial stresses when applying axial stresses. The radial stress is assumed to be $\sigma_{r}=k \sigma_{a} k=v /(1-\mathrm{v})$, where V is the Poisson ration of
the hardcore. The Poisson ratio is equal to 0.22 for Tungsten Carbide. Thus $\mathrm{k}=$ 0.285 . The component of the radial stress normal to the contact area gives a stress equal to $\sigma_{a} k \operatorname{Cos}(\alpha)$. The total normal stress to the surfacebecomes
$\sigma_{n}=\sigma_{a} \operatorname{Sin}(\alpha)+\sigma_{a} k \operatorname{Cos}(\alpha)$, The shear stress is $\sigma_{s}=\mu \sigma_{n}$, to read

$$
\begin{equation*}
\sigma_{n}=\sigma_{a} \operatorname{Sin}(\alpha)+\sigma_{a} k \operatorname{Cos}(\alpha), \sigma_{s}=\mu \sigma_{n}=\sigma_{a} \mu(\operatorname{Sin}(\alpha)+k \operatorname{Cos}(\alpha)) \tag{5}
\end{equation*}
$$



Figure 2: Hardcore with conical base.

The axial stress will vary along the axial direction in the conical part due to friction. Assume as an approximation that the stress is constant and equal to $m a /\left(\pi R^{2}\right)$ This should be a good approximation as long as the conical part is small compared to the length of the hardcore. The torque becomes for a cone with radiuses $r_{0}$ and $R$

$$
\begin{align*}
& T=\int_{S} \sigma_{S} x r d S=\frac{2 \mu m a R}{3}\left(1-\frac{r_{0}^{3}}{R^{3}}\right)\left(1+\frac{k}{\operatorname{Tan}(\alpha)}\right)=\left(\frac{2 \mu m a R}{3}\right) t(x(\alpha)), \\
& \operatorname{Tan}(\alpha) \stackrel{\text { def }}{=} x(\alpha) t(x) \stackrel{\operatorname{def}}{=}\left(\frac{x+k}{x}\right)\left(1-(1-x \beta)^{3}\right)=\beta(x+k)\left(3-3 \beta x+\beta^{2} x^{2}\right), x \neq 0 \tag{6}
\end{align*}
$$

We have set that set that $r_{0}=R$ - $\operatorname{Tan}(\alpha) H_{c}$ and $\beta \stackrel{\text { def }}{=} H_{c} R$ where $H_{c}$ is the height of the conical part. Eq. (6) gives that for a given $r_{0}$ the half angle should be as small as possible to achieve the largest torque. If $k$ equals zero, the angle dependency cancel out and the torque is dependent on the integral over the projected contact area in the axial
direction. When $r_{0}=0$ and $\mathrm{k}=0$, we thus obtain the same result as for a cylinder with flat base. $\mathrm{t}(\mathrm{x})$ is a torque factor that relates to the geometry of the cone.

In Figure 3 we have plotted the torque factor $\mathrm{t}(\mathrm{x})$ as a function of the cone angle in degrees.


Figure 3: The torque factor $\mathrm{t}(\mathrm{x})$ as a function of the cone angle for materials with different Poisson ratios, in the care where $\mathrm{Hc} / \mathrm{R}=1$

The k for Tungsten Carbide equals 0.28 . We observe that the maximum value is achieved for a half angle around 30 degrees, which is the angle on the chosen design. This gives the largest torque.

## THE RIFFLING PITCH IN THE GUN BARREL

Consider a Gun barrel where the twist varies along the gun barrel. Say that the position of a rifle is given by the helix

$$
\begin{equation*}
\vec{r}(\vartheta)=\frac{D}{2}(\vec{i} \operatorname{Cos}(\vartheta)+\vec{j} \operatorname{Sin}(\vartheta))+\vec{k} \mathbf{z} \vec{r}(\vartheta) \tag{7}
\end{equation*}
$$

where D is the Calibre, $z(\vartheta)$. is the distance along the axial direction as a function of the twist angle $\vartheta$. of the projectile. The position is measured from the initial position of the driving band on the projectile. The angle between the tangential vector of a rifle and
the z axis (riffling pitch) is given by $\theta$, to read

$$
\begin{equation*}
\theta=\operatorname{ArcCos}\left(\frac{z^{\prime}(\vartheta)}{\sqrt{(D / 2)^{2}+z^{\prime}(\vartheta)^{2}}}\right) \approx \frac{(D / 2)}{z^{\prime}(\vartheta)},(D / 2) \ll z^{\prime}(\vartheta) \tag{8}
\end{equation*}
$$

There is a one to one relation between the twist angle and the axial position, say $\vartheta=\vartheta(\mathrm{z})$. The angular velocity and the angular acceleration then becomes

$$
\begin{equation*}
\omega=\frac{\partial \vartheta}{\partial t}=\frac{\partial \vartheta}{\partial z} \frac{\partial z}{\partial t}, \dot{\omega}=\frac{\partial^{2} \vartheta}{\partial t^{2}} \frac{\partial^{2} \vartheta}{\partial z^{2}}\left(\frac{\partial z}{\partial t}\right)^{2}+\frac{\partial \vartheta}{\partial z} \frac{\partial^{2} z}{\partial t^{2}}=\frac{\partial^{2} \vartheta}{\partial z^{2}} v^{2}+\frac{\partial \vartheta}{\partial z} \tag{9}
\end{equation*}
$$

$v(z)$ is the velocity of the projectile. Eq. (4) and eq. (9) then gives

$$
\begin{equation*}
\dot{\omega} \leq \dot{\omega}_{\max }=\frac{4 \mu \alpha}{3 R} \Rightarrow \frac{\partial \vartheta}{\partial z} \leq \frac{4 \mu}{3 R}-\frac{\partial^{2} \vartheta v^{2}}{\partial z^{2}} \frac{v^{2}}{\alpha},: \text { No malfunction } \tag{10}
\end{equation*}
$$

We observe in eq. (10) the quite obvious result that for progressive twisting, i.e. $\partial^{2} \vartheta / \partial z^{2}>0$, the condition for no malfunction is harder to fulfil, all other conditions being equal.

In addition, inserting the formula in eq. (8) for the riffle angle between the tangential vector and the z axis gives that eq. (10) can be written approximately as

$$
\begin{equation*}
\theta \leq \frac{2 \mu D}{3 R}-\frac{\partial \theta v^{2}}{\partial z} \frac{v^{2}}{\alpha} \tag{11}
\end{equation*}
$$

This means that the rifling pitch, $\theta$, must be smaller than the right hand side of eq. (11) to achieve spin up of the hard core.

The general case for a hardcore with a cone can be achieved mathematical by letting
$\mu \rightarrow \mu\left(1-r_{0}^{3} / R^{3}\right)(1+k / \operatorname{Tan}(\alpha))$ in eq. (11). Eq.- (11) will be used in the next sections to study that no malfunction takes place.

## THE EQUATION OF MOTION OF THE PROJECTILE

The pressure $p_{b r}$ at the breech of the gun barrel was measured. The base pressure $b_{p}$ on
the projectile is calculated to be used in the equation of motion of the projectile. We apply that the velocity is a linear function of the axial position and that the density is constant along the axial position. It follows that

$$
\begin{equation*}
p_{b}=\left(p_{b r}+\frac{M}{2 m_{p}} p_{r s}\right),\left(1+\frac{M}{2 m_{p}}\right) \tag{12}
\end{equation*}
$$

M is the mass of gun barrel gases and propellant and $m_{p}$ is the mass of the projectile. The equation of motion for the projectile is $m_{p} \alpha=\left(p_{b}-p_{r s}\right) A$, where $p_{r s} A$ is a force on the projectile due to friction between the projectile and the gun barrel and due to air resistance. A is the inner reference area of the gun barrel. The base pressure is used for the calculations in the next section.

## SIMULATION RESULTS

The rifling pitch is as an input function given in radians as a function of the position in the gun barrel. The square of the velocity divided by the acceleration, both as a function of position of the projectile in the gun barrel, are found from the equation of motion of the projectile, to read

$$
\begin{align*}
& \theta(s)=\operatorname{Max}[0,0.138(1-\operatorname{Exp}(-1.546(\mathrm{~s}-0,072)))],[\mathrm{s}]: \text { meter } \\
& v^{2}(s) / \alpha(s)=17.38\left(1-\operatorname{Exp}\left(1-0.2944 s^{2}\right)\right) \tag{13}
\end{align*}
$$

The simulated exit velocity of $1129 \mathrm{~m} / \mathrm{s}$ was in good agreement with the experimental recording of $1136 \mathrm{~m} / \mathrm{s}$. We neglected the friction between the projectile and the gun barrel. The air resistance contributes marginally to the solution. A factor that gives variation of $+-10 \mathrm{~m} / \mathrm{s}$ is the necessary pressure build up before the projectile can leave the cartridge. This pressure was set to 60 MPa based on measurements of the force needed to remove the projectile from the cartridge. The coefficient of friction was further set to 0.1 . Figure 4 shows the rifling pitch (red) of the barrel and the right hand side of eq. (11) with a hardcore with cone (black) and a hard core with flat base (green) as a function of the position in the gun barrel. We observe that the design with the cone is easier to spin up compared to the design with a flat base; moving from the green to the black curve the right hand side in eq. (11) is significantly enlarged. Without the
cone, the right hand side of eq. (11) is marginally below the rifling pitch at position 1.0-1.25 meter from the cartridge. Thus the hardcore with a flat base will not spin up with the rest of the projectile between 1.0-1.25 meter. The projectile has been fired with the chosen cone and no malfunction is observed. Figure 4: The threshold value for the rifling pitch given by eq. 11 as a function of the position inside the gun barrel.


## CONCLUSIONS/DISCUSSION

We have analyzed some inertial ballistic requirement to achieve spin up of a hardcore fitted into a full bore projectile. The objective was to study the conditions required when frictional forces are used to spin up the hardcore. We have found analytical and numerical solutions, which have been used to analyze different geometrical structures of the hardcore. A hardcore with a conical base is a better design with respect to spin up compared to a hardcore with a flat base due to the Poisson ratio of the hardcore material.

