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BALLOTING IN A FRENET FRAME

Satoru Shoji

Wakayama-dai 2-2-21-104, Shimamoto-cho, Osaka, 618-0024, Japan email:shojistr@ybb.ne.jp

A study was undertaken to formulate "In-Bore Balloting" which plays leading role in a dynamic process towards projectile jump. By way of equivalent transformation from the barrel deflection to the coordinate acceleration, constraint on balloting motion were largely simplified, and at the same time, continuity and common use over interior and exterior area were secured. This paper presents formulae of angular and translational accelerations for the new coordinate system, starting from barrel-deflection and in-bore traveling.

INTRODUCTION

Projectile jump appears complicated phenomena in its factor contribution. Many have tried to find out the relation between specific factors and resulting jump but in vain. It is regrettable to admit encyclopedia:Wikipedia who wrote "Transitional ballistic is not an exact science". To analyze the jump problem, we need two side approach: one is the phenomenological consideration based on unusual features of jump, the other is the numerical reproduction of high variability of jump with simple model calculation.

Poor reproducibility must be the true nature of jump. It would be better for us to start with the generally accepted information and experiments with unexpected results as shown in Table 1. It is evident that the real problem of Transitional Ballistics is the uncertainty or variability of jump and not the jump itself. Any jump model shall be

Table 1. Features of Jump

1. Jump would generate to any direction and on any type of projectile [1]
2. "Occasion to Occasion Error" .
3. The first shot usually shows unexpected point of impact.
4. Gap between sabot and rod has no relation to the magnitude of jump.
5. Larger clearance between sabot and barrel decreases the variability of jump [2].
7. There are quite contradictory experiments on the effect of band stiffness. [2].

capable to explain how these phenomena would occur. We will speculate a scenario of generating jump in Table 2. considering these features. An emphasis was laid on the change in attitude of projectile while exiting the muzzle. It means variability of jump comes from the **phase shift** of balloting (= phase shift concept). Any other factors are the “prepositional conditions” for the phase shift.

Table 2. A Scenario to Generate Jump

1. On firing, the barrel generates lateral **vibration** [3].
2. Repeated collides of projectile against barrel make **balloting**.
3. While exiting the muzzle, the projectile that **looses a support** at forward bourrelet changes its angle drastically especially when it collides against barrel at rear bourrelet.
4. Angular velocity given at the muzzle transforms itself to aerodynamic jump [4], and the translational velocity becomes lateral jump, Addition of the two makes **primary jump.**
5. The primary jump will be **augmented** by asymmetric forces which may come from muzzle blast and sabot separation.
6. Bending force stored in projectile would be the cause of additional **flexural** jump.[5]

The second approach is the model calculation of balloting and of change in attitude at muzzle. We propose a new coordinate system where equation of motion is universal over internal /external area, named: “**Barrel Coordinate System**”.

Definition of Barrel Coordinate System

The Barrel coordinate system shares the same orthogonal axis as of Frenet frame that goes along a space curve. The space curve is the trajectory of point C(s) that is the osculation on the instantaneous centerline of the barrel, and the center of projectile is in the normal plane at the same instance (See Figure 1).

Barrel coordinate system has bonus benefits; 1) Barrel with deflections either dynamic or static can be treated as a straight cylinder. 2) Exterior area can be treated as an imaginary interior area, and clarify the muzzle brake interferences.

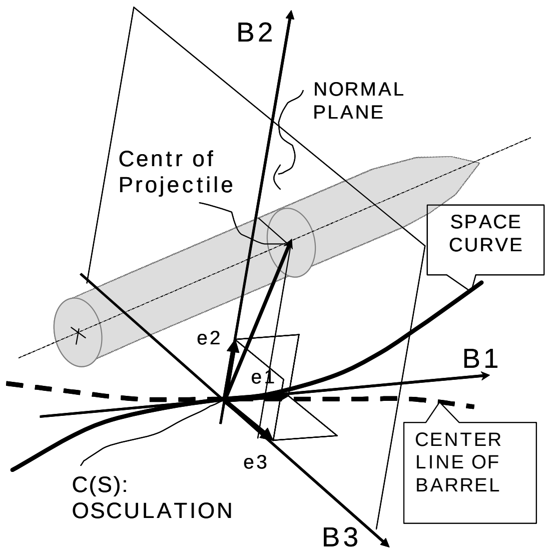


Figure 1.
B_1-B_2-B_3 Barrel Coordinate System
e_1-e_2-e_3 :Frenet Frame Basis

ANALYSIS

Irreversible flow of jump calculation is shown in Figure 2.

Augmented Jump is estimated as much as one third of the projectile jump. This is the problem of the chemistry and fluid dynamics, and could be argued separately.

Flexural Jump comes from aerodynamics of bending projectile, but has no direct effect on primary jump. The magnitude is estimated to be +/- 0.25 mrad at most [5]

Primary Jump is derived from eq.(1). MAV and MCV are the outputs of the motion calculation of the projectile. This kinetic system is constrained with unusual condition that projectile is accelerated in a vibrating tube with small clearances.

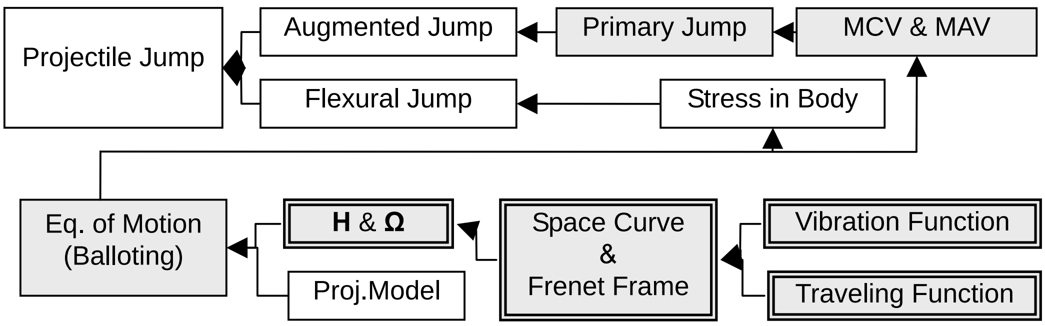


Figure 2. Flow of Jump Calculation

MAV : Angular velocity of projectile after the muzzle
MCV : Lateral velocity of projectile after the muzzle
H **:** Translational acceleration of the Barrel coordinate system
Ω : Angular acceleration of the Barrel coordinate system

(Primary jump)
$$B_{JMP} = \left(MCV - B_D \cdot MAV \cdot d_0\right)/Vo \tag{1}$$

$$B_D = \left(\frac{C_{NA} - C_D}{C_{NA} \cdot \Delta L}\right)\frac{It}{m \cdot d_0^2} \quad \text{--- (Exhibit A-1)} \tag{2}$$

V_0 Muzzle velocity, d_0:Rod diameter, m :Mass of the projectile, It**:**Moment of inertia around center of mass, C_{NA}:Normal force coefficient, C_D :Drag coefficient, ΔL :Separation between center of mass and center of aero force in rod dia unit.

Projectile Model: Any type of model is available so long as its lateral spring properties were evident

Equation of Motion in the accelerated coordinate system has the ordinary form except the second term that is isolated from the contact forces.

(Translational Motion)
$$\ddot{r}^i = \frac{\sum_j F_j^i}{m^i} - H_2 \tag{3}$$

(Rotational Motion)
$$\ddot{\theta}^i = \frac{\sum_j N_j^i}{I^i} - \Omega_2 \tag{4}$$

where,

H_2 translational acceleration function of Barrel coordinate system (vector) ---- eq.(16)

Ω_2 angular acceleration function of Barrel coordinate system (vector) ---- eq.(18)

i superscript refers to body i

(Further explanations were skipped for other common symbols.)

Frame Acceleration H2 and Ώ2

[Vibration Function: eq. (5)]

The function, values of βn, ωn are derived from beam theory and dimensions of barrel assuming that the barrel is **"a beam with two free supports and one free end",** while ai_n are empirical coefficients to fit live firing data which may change with the propellant charge, mass of the moving parts of the gun system.

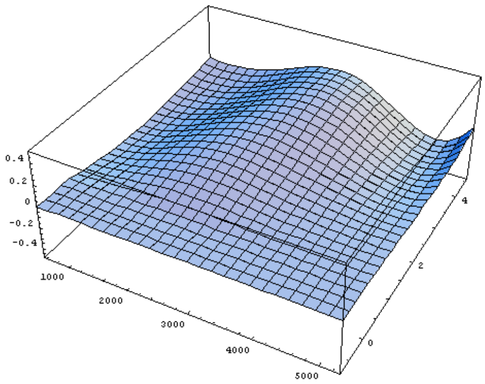


Figure 3. Vibration Function

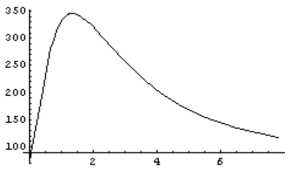
$$Y_G(x,t) = \sum_{n=1}^{3} \{a_{1n}(\cos\beta_n x - \cosh\beta_n x) + a_{2n}\sin\beta_n x + a_{3n}\sinh\beta_n x\}\sin\omega_n t \tag{5}$$

[Traveling Functions]

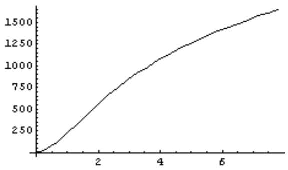
The following formulae were picked up for convenience of integration.

$$v(t) = a_3 t\left\{Pss + \frac{a_1(4a_2 + t)t^2}{12a_2{}^2(a_2 + t)^4}\right\} \tag{6}$$

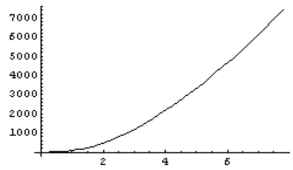
$$x(t) = a_3\frac{t^2}{2}\left\{Pss + \frac{a_1 t^2}{6a_2^2(a_2 + t)^3}\right\} \tag{7}$$



acceleration: a(t)



velocity:v(t)



travel:x(t)

[Circular Vibration]   We will get a 3D trajectory of a projectile as a intersection of two surfaces :vibration function vs. travel-time relation. This will make a vector function:$Y_G y(x,t)$.

In order to evaluate the effect of the slightest deflection of barrel , **arc length parameter: "s"** were used instead of travel parameter: "x".

Relation: t = t(x) reduces time parameter from eq.(7) and get $Y_G y(s)=Y_G(x,t)$.

Supposing the circular vibration mode, $Y_G z(s)$ were taken into account as a perpendicular component of vibration, where zap(amplitude reduction rate), dsz(starting delay of vibration) were arbitrarily taken.

$$Y_G z(s) = zap Y_G y(s - dsz) \tag{8}$$

[Space Curve] Now we have C(s): a vector function in inertial coordinate system. This is **A Space Curve** in differential geometry.

$$C(s) = \{s, Y_G y(s), Y_G z(s)\} \tag{9}$$

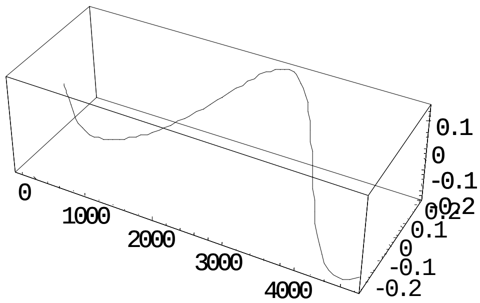
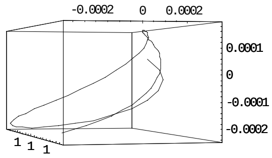


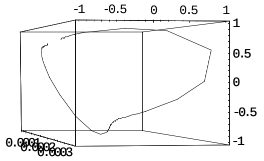
Figure 4. The space Curve:C(s)
(Deflection vs. Distance in mm)

[Basis of Frenet frame]   Some characteristic values of the space curve were given by the formulae, where, e_1:tangent vector, e_2:principal normal vector, e_3:bi-normal vector, and "dot" denotes the time derivative of C(s(t)).

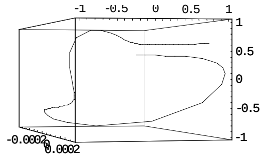
$$e_1 = \dot{C} \Big/ \left\| \dot{C} \right\| \tag{10}$$



$$e_2 = (\dot{C} \times \ddot{C}) \Big/ \left\| \dot{C} \times \ddot{C} \right\| \tag{11}$$



$$e_3 = e_1 \times e_2 \tag{12}$$



[Curvature and Torsion of the Space Curve]

(Curvature) $$\kappa(s) = \left\| \dot{C} \times \ddot{C} \right\| \Big/ \left\| \dot{C} \right\|^3 \tag{13}$$

(Torsion) $$\tau(s) = \det(\dot{C}, \ddot{C}, \dddot{C}) \Big/ \left\| \dot{C} \times \ddot{C} \right\|^2 \tag{14}$$

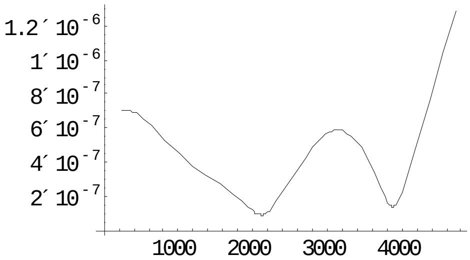


Figure 5. Curvature

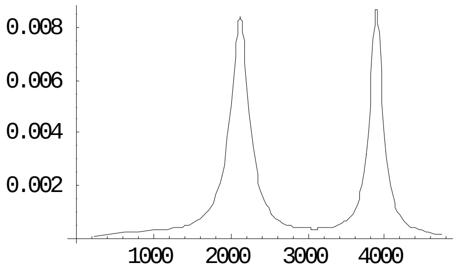


Figure 6. Torsion

These curvature/torsion feature would governs the balloting phase together with the lateral spring characteristics of the projectile model.

[Velocity and Acceleration of Frenet Frame] (Exhibit A-2 for proof of eq.(15)~(18))

H_1, H_2 :translational velocity & acceleration,
Ω_1, Ω_2 :angular velocity & acceleration

ν_s : speed along the space curve $\nu_s \equiv ds/dt = \|\dot{C}\| \cong v(t)$:

$$H_1 = \nu_s e_1 \tag{15}$$

$$H_2 = \nu_s^2 \kappa e_2 \tag{16}$$

$$\Omega_1 = \nu_s \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \quad \text{(modified Frenet-Serret formula)} \tag{17}$$

$$\Omega_2 = -\nu_s^2 \begin{pmatrix} \kappa^2 & -\kappa' & -\kappa\tau \\ \kappa' & \kappa^2 + \tau^2 & -\tau' \\ -\kappa\tau & \tau' & \tau^2 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \tag{18}$$

$e_1\Omega_2$, $e_2\Omega_2$ and $e_3\Omega_2$ are the components of angular acceleration vector of the Barrel coordinate system in directions of rolling, yawing and pitching, respectively.

DISCUSSIONS

Frenet-Serret formula in differential geometry was found to have, in kinematics, the meaning of angular velocity of a frame. We have extended this finding to a new formula eq.(18) that presents the angular acceleration of Barrel coordinate system for our equation of motion. 3D balloting was made simple and plain by this transformation. To complete the jump calculation, we have to specify the model of projectile, however, any type of model is applicable to the equations of motion :eq.(3) and eq.(4).

We had once computed 2D jump under the same scenario as of Table 2 [6], and found that the feature of jump in Table1 were reasonably explained by **the phase-shift concept**. The situation is the same in 3D case. We cannot point out a particular factor that makes variability less, we can, however, categorize the dominant factors on high variability in Table 3. Degree of these material effects will be evaluated in a separate paper by giving a specific projectile dimensions and under specified conditions.

Table 3 Dominant Factors on High Variability of Jump

Group I :Inadequate **design** of the last collision point too close to the muzzle.
1a: Elasticity of components. 1b: Moment of inertia of heavy rod,
1c: Metrics such as spun between supports and clearances.
2a: Barrel support positions, 2b: Weight and length of barrel.
2c: Cracking pressure of recoil start, 2d: Center of mass of recoil parts,

Group II :poor **quality** that shifts the balloting phase with ease.
1 (Unsteady pressure rise),
1a: Poor ignition of propellant, 1b: Unsteady burning rate,
2 (Variation of “shot start pressure ”),
2a: Stiffness and diameter of the sealing band, 2b: Excessive erosion at forcing cone.
3 (Variation in start of carriage movement).
3a: Break of lubricant film at sliding surface. 3b: Excessive distortion at mating parts.

CONCLUSION

New wine called Transitional Ballistics has found her new bottle called Ballel Coordinate System that is based on the equivalent transformation from the curved space to the coordinate accelerations using the space curve theory of differential geometry. We believe in “raison d’etre” of transitional ballistics in the field of science and technology.

This analysis is not completed. Further conjugation with small experiments such as vibration test would be useful for improving the balloting analysis.

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[3] H.Reuschel, p196-p198, Handbook on Weaponry, Rheinmetall Industriewerburg.GmbH.(1982)
[4] Mark Bundy, The Regional Nature of Aerodynamic Jump, ARL-TR-1872 (1999)
[5] E.M.Schmidt, Aerodynamic Jump of A Flexing Projectile, 20th International Symposium on Ballistics, (Orland, FL 2002)
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On Demand Exhibits:

[A-1] :Bundy Constant

[A-2] :Proof of Formulae for Coordinate Acceleration