MODELING OF NORMAL PERFORATION OF REINFORCED CONCRETE SLAB BY RIGID PROJECTILE*

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An analytical model on the normal perforation of reinforced concrete slabs is constructed in the present paper. The effect of reinforcing bars is further hybridized in a general three-stage model consisting of initial crater, tunneling and shear plugging. Besides three dimensionless numbers, i.e., impact function *I*, geometry function of projectile *N* and the dimensionless thickness of concrete target χ, which are employed to predict the ballistic performance of perforation of plain concrete slabs, the reinforcement ratio ρ_s of concrete and the tensile strength f_s of reinforcing bars are considered as the other main factors of influencing the perforation process. Simpler solutions of ballistic performances of normal perforation of reinforced concrete slabs are formulated in the present paper. Theoretical predictions agree well with individual published experimental data.

INTRODUCTION

Penetration of reinforced concrete has been receiving increased attention. Different from plain concrete that only the strength dominates its penetration resistance, reinforced concrete may be influenced by both the concrete strength and the amount of reinforcement. The shape and depth of craters depend on the layout and embedding depth of reinforcing meshes. The other factors, such as material properties and diameter of reinforcing bars, mesh size and space, also affect the final results of perforation.

For engineering models, most empirical formulae didn't include the effect of reinforcing bars on penetration/perforation, although considerable test results came from the impacts of reinforced concrete. Only in individual cases, e.g. Barr[1], the effect of reinforcing bars had been considered in the perforation formulae which mainly fitted to the experimental data. Theoretical modelling and numerical simulation on the penetration/perforation of reinforced concrete are usually conducted by means of ignoring reinforcing bars, or being equivalent to strength-enhanced concrete, and even being equivalent to the sandwich structure composed of concrete and thin armour plate.

Although not applied into a formula to predict the perforation resistance, Riera [2] suggested a way to include reinforcement in a perforation formula, i.e., based on the contribution of reinforcing bars to the tensile strength of concrete. Dancygier [3] proposed a model to evaluate the effect of reinforcement ratio on the perforation resistance of reinforced concrete quantitatively. Similar to Riera's suggestion, the equivalent tensile capacity depends on the concrete tensile strength and on the amount and yield stress of reinforcement. A theoretical expression, which includes the reinforcement ratio as a variable, is included in the existing perforation formulae.

In the present paper, an alternative modelling of normal perforation of reinforced concrete subjected to rigid projectile impact is proposed. A general three-stage model consisting of initial crater, tunnelling and shear plugging after Chen et al.[4] is employed, and hybridized with the effect of reinforcing bars. Reinforcement ratio ρ_s of concrete (or area density) and the tensile strength f_s of reinforcing bars are considered as the main factors of reinforcement influencing the perforation process, and are introduced together in a dimensionless number Θ of reinforcement. The present model has higher degree of accuracy and simpler formulae than Dancygier's model [3].

ANALYTICAL MODEL

Consider a rigid projectile with mass M, diameter d and a general convex nose shape impacting a reinforced concrete target (thickness H) at initial velocity V_0, and penetrating through the concrete target medium at rigid-body velocity V. The process of perforation has two possible scenarios. One is a complete perforation, which includes the initial cratering, tunnelling and rear cratering (see Fig.1). The other is initial cratering immediately followed by shear plugging, i.e., the tunnelling stage is omitted (see Fig.2). Recurring to the area density ρ_s of reinforcement, all the reinforcing bars are localized in the rear cratering of deformation. When a failure surface (rear cratering) occurs, it simultaneously activates a bowl action of the reinforcement.

DYNAMIC CAVITY EXPANSION THEORY

In the process of normal penetration through a concrete target, the initial crater is assumed to be a conical shape having an axial depth kd. The axial resistant forces on the

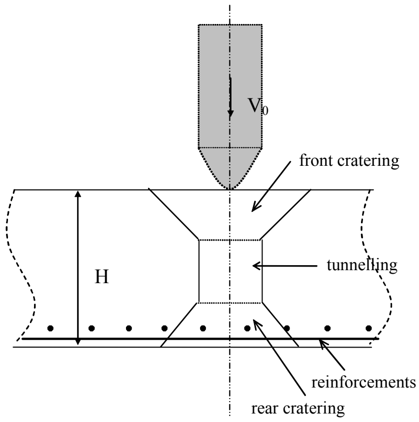

Figure 1. Case of thick reinforced concrete target

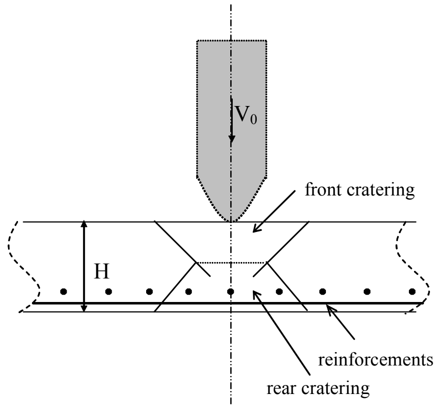

Figure 2. Case of thin reinforced concrete target

projectile nose during the initial cratering and penetration are, respectively [5],

$$F = c\frac{x}{d} \quad \text{for } \frac{x}{d} \le k\,, \tag{1a}$$

$$F = \frac{\pi d^2}{4}\left(Sf_c + N^* \rho V^2\right) \quad \text{for } \frac{x}{d} \ge k\,, \tag{1b}$$

where x denotes the instantaneous penetration depth, ρ is the density of concrete and S is an empirical constant related to the unconfined compressive strength of concrete f_c. Impact function I, geometry function N and nose shape factor N^* are given by [4]

$$I = \frac{MV_0^2}{d^3 Sf_c}\,,\ N = \frac{M}{\rho d^3 N^*} \ \text{ and } \ N^* = -\frac{8}{d^2}\int_0^h \frac{yy'^3}{1+y'^2}dx \tag{2}$$

where y is the geometric definition of projectile's nose curve. Previous studies showed that I and N are the two dominating factors in the penetration process.

SHEAR PLUGGING CRITERIA

In normal impact, the rear crater can be approximately considered as a cone-shaped plug with a cone slope angle α. The failure stress in pure shear is defined as $\tau_f = f_c/\sqrt{3}$. As concrete is a brittle material, it is assumed that the plug is separated

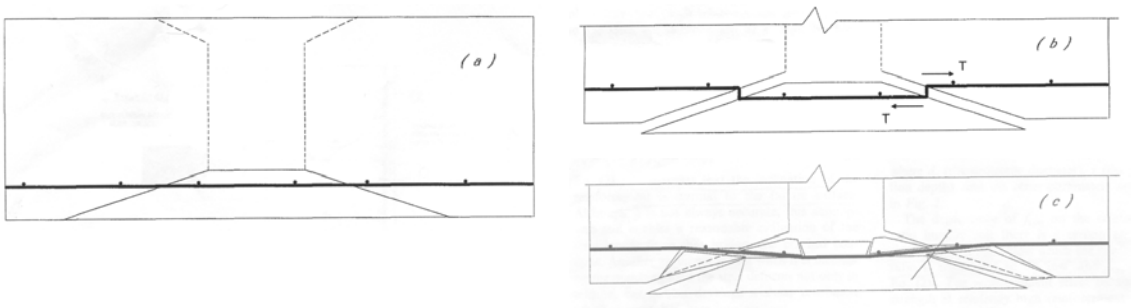

Fig. 3 Schematic description of the failure process at the rear face [3]

from the surrounding material as soon as the shear failure criterion is satisfied along the plug surface. Regarding to the reinforced concrete, the separation of rear plug from the concrete matrix should also include the tensile failure of reinforcing bars (see Fig. 3).

A dimensionless number is introduced,

$$\Theta = \sqrt{3}\chi\rho_s \sin\alpha f_s / f_c \tag{3}$$

which simultaneously related to the reinforcement ratio ρ_s and the tensile strength f_s of reinforcing bars. χ=H/d is the dimensionless thickness of concrete target. Shear plugging occurs as soon as the axial resistant force reaches a critical value of

$$F = \frac{1}{\sqrt{3}} f_c A_s \cos\alpha \left(1 + \Theta \frac{d}{H^*}\right) \tag{4}$$

in which, A_s is the shear area of the conical plug surface[4],

$$A_s = \frac{\chi\pi d^2}{\cos\alpha} \frac{H^*}{H} \left(1 + \chi \frac{H^*}{H} \tan\alpha\right) \tag{5}$$

Θ=0 represents the case of plain concrete [4]. Eq.(4) is employed to solve the residual thickness H^* of cone-shaped plug. Obviously, A_s and H^*/ H are independent of initial velocity V_0 and can be determined by the geometric configuration of perforation.

BALLISTIC PERFORMANCE OF NORMAL PERFORATION

For the thick concrete panels, we assume V_* to be the velocity of projectile at the end of tunneling, which is determined by the dynamic cavity expansion theory and plug model. For the thin concrete panels, V_* is the velocity of projectile at the transitional instant from initial cratering to shear plugging. Essentially, the plug always disintegrates into fragments, which caused by the tensile stresses arising from the reflections of stress wave prior to shear plugging. Thus we regard V_* as the residual/exit velocity V_r of projectile after perforation. The ballistic limit is obtained when $V_*=I_*=0$. We assume V_{BL} as the ballistic limit and its corresponding impact function as $I_{BL} = MV_{BL}^2/(d^3Sf_c)$, and H_{BL}^* as the corresponding value of H^* at the ballistic limit, respectively.

For normal perforation with $N >> I$ and $N >> 1$, which are common in practice associated with sharp and slender projectiles, simplified formulae for ballistic performance of perforation of reinforced concrete can be deducted as follows,

$$V_{BL} = \sqrt{\frac{\pi d^3 Sf_c}{4kM}}\left(\chi - \frac{H_{BL}^*}{d}\right), \text{ for } \chi \le \chi_c \tag{6a}$$

$$V_{BL} = \sqrt{\frac{\pi d^3 Sf_c}{2M}} \cdot \sqrt{\left(\chi - \chi_c + \frac{k}{2}\right)}, \text{ for } \chi > \chi_c \tag{6b}$$

The dimensionless critical thickness χ_c of concrete target, dominated by f_c and Θ, is introduced to classify the thin panels from the general reinforced concrete targets, at which the tunneling process can be ignored critically.

$$\chi_c = \frac{H_c}{d} = \frac{\sqrt{(1+\Theta\tan\alpha)^2 + \left(\sqrt{3}S - 4\Theta\right)\tan\alpha} - (1+\Theta\tan\alpha)}{2\tan\alpha} + k \tag{7}$$

When $V_0 > V_{BL}$, the residual velocity of projectile can be formulated

$$V_* = (V_0 - V_{BL}), \text{ for } \chi \le \chi_c, \tag{8a}$$

$$V_* = \sqrt{\left(V_0^2 - V_{BL}^2\right)}, \text{ for } \chi > \chi_c. \tag{8b}$$

In engineering practice, the perforation limit e for a reinforced concrete target is defined as the minimum thickness of the target to resist projectile perforation, i.e., the minimum target thickness without perforation (or without the occurrence of plugging in the present case). Therefore, e can be determined by $e/d = X/d + H_{BL}^*/d$ or

$$\frac{e}{d}=\frac{X}{d}+\frac{\sqrt{(1+\Theta\tan\alpha)^2+\left[\sqrt{3}S\left(\frac{X}{kd}\right)-4\Theta\right]\tan\alpha}-(1+\Theta\tan\alpha)}{2\tan\alpha},\ \text{for}\frac{X}{d}\le k \quad (9a)$$

$$\frac{e}{d}=\frac{X}{d}+\frac{\sqrt{(1+\Theta\tan\alpha)^2+\left(\sqrt{3}S-4\Theta\right)\tan\alpha}-(1+\Theta\tan\alpha)}{2\tan\alpha},\ \text{for}\ \frac{X}{d}>k. \quad (9b)$$

It is worthwhile to indicate that, if Θ=0, all the above formulations reduce to the scenario of normal perforation of plain concrete slabs by rigid projectile, and much simpler solutions of ballistic performance than Li and Tong [6] and Chen et al. [4] can be formulated from Eqs.(6-9) in the present paper.

EXPERIMENTAL ANALYSES

Figs.4 and 5 plot the residual velocity versus initial impact velocity from the test results on NSC and HSC targets after Hanchak, et al. [7] and the corresponding predictions by present model, which include two cases of either considering or ignoring the reinforcement. According to the present analysis, the ballistic limits increase while the residual velocities decrease a little due to the effect of reinforcement. However, since the targets are all light reinforced concrete and the reinforcement ratio ρ_s is only 0.5%, the change of terminal ballistic performance is not notable.

Quantitatively, after [3], Fig.6 and Fig.7 plot the influence of different

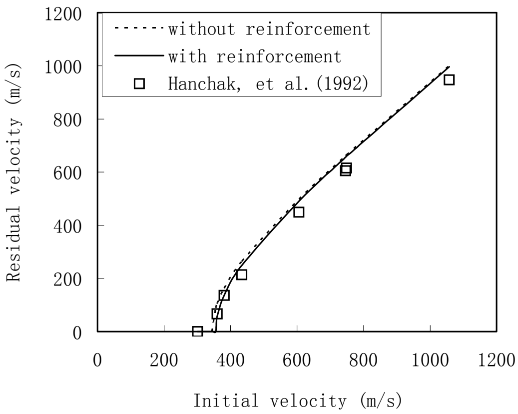

Figure 4. Prediction of residual velocity and test data of NSC

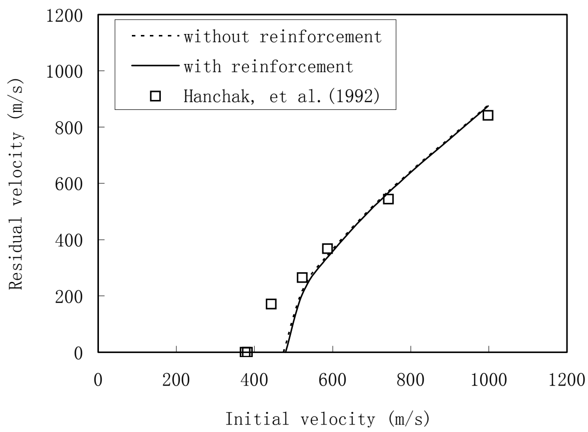

Figure 5. Prediction of residual velocity and test data of HSC

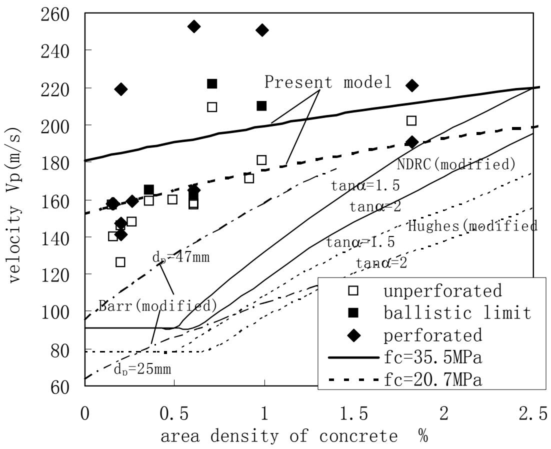

Figure 6. Perforation velocity vs reinforcement ratio regarding to 50mm thick targets

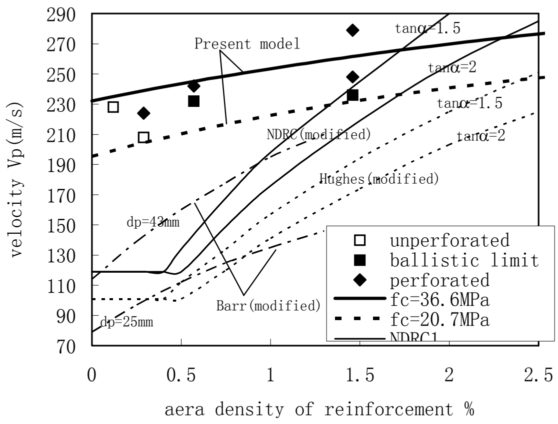

Figure 7. Perforation velocity vs reinforcement ratio regarding to 60mm thick targets

reinforcement ratios on the perforation velocities of the 50mm and 60mm thick concrete panels, respectively. It includes the test results and the predictions by the present model, as well as NDRC, Hughes, and Barr equations. As can be seen in Fig.6 and Fig.7, the experimental perforation velocities are higher than the curves of the empirical formulae, but are very close to the curves of present modelling. Therein the present model shows a more reasonable prediction than the mentioned empirical formulae. Qualitatively, the higher the reinforcement ratio, the higher the perforation velocity or the element perforation resistance, and vice versa. The present model has higher degree of accuracy and simpler formulae than the model suggested by Dancygier [3].

The variations of the ballistic limit with concrete strength f_c are plotted in Fig.8 for different reinforcement ratios regarding to the concrete panels of [3]. It shows that ballistic limit increases with either concrete strength or reinforcement ratio, and thus both reinforcement and concrete strength play important role in the perforation resistant ability. Fig. 9 plots the influence of the dimensionless number Θ on the dimensionless height H_{bl}^{*}/d of rear conical plug. Accompanied with increasing Θ, i.e., increasing the values of ρ_s or f_s, H_{bl}^{*}/d drops sharply. It shows the reinforcement may obviously reduce the volume of rear cratering, which expected to limit the damages zone depth and the size of the ejected concrete fragments.

CONCLUSIONS

The effect of reinforcing bars is hybridized in a general three-stage model to study

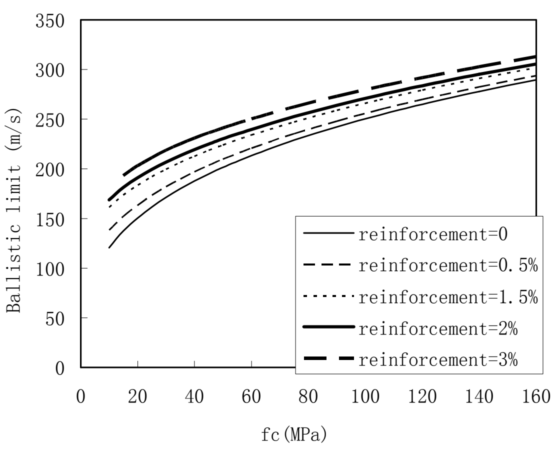

Figure 8. The ballistic limit vs f_c under different reinforcement ratios

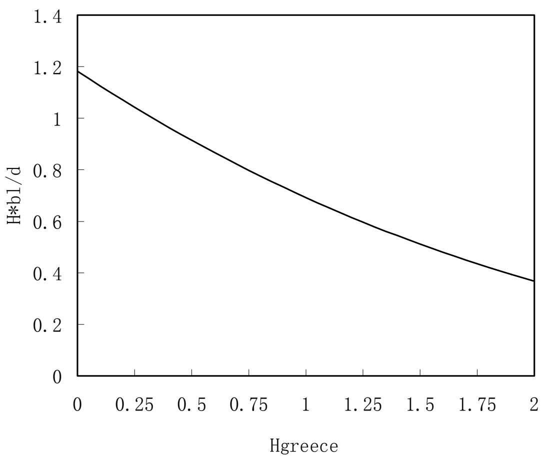

Figure 9. Variation of H_{bl}^{*}/d with Θ

the normal perforation of reinforced concrete subjected to projectile impact. Total four dimensionless numbers, i.e., I, N and χ as well as the reinforcement number Θ, dominate the whole normal perforation of a reinforced concrete. Explicit ballistic performances of reinforced concrete are formulated. Predictions agree well with published experimental data and have higher degree of accuracy than Dancygier [3].

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