NUMERICAL-ANALYTICAL MODEL OF PENETRATION OF LONG ELASTICALLY DEFORMABLE NON-HOMOGENEOUS PROJECTILES INTO SEMI-INFINITE TARGETS

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A new numerical-analytical model of penetration of long axisymmetric elastically deformable non-homogeneous projectiles in semi-infinite targets is presented. A background of this model is the integral-differential equation of ballistics for non-deformable projectile. This equation is obtained on the basis of Lagrange-Cauchy integral for non-stationary irrotational motion of incompressible fluid, and equations of expansion of spherical cavity. The field of velocities in target is defined by real projectile shape and a shape of entering hole. The elastic deformations of projectile associated with its forced longitudinal oscillations are taken into account. Oscillation patterns for homogeneous and non-homogeneous projectile are calculated and compared. The results are compared with known experimental and calculated data.

INTRODUCTION

The investigation of interaction of rigid penetrator and targets of different nature is going on for a quite a long time. However, simple analytical models adequately describing experimental results still continue to improve [1,2]. These improvements are achieved by taking into account of some factors and discarding other. Among such factors are friction, wear, elastic deformation, blunting, yaw, oscillations, to name few.

A recent study of penetration of homogeneous rigid and elastically deformable projectiles into semi-infinite targets from different materials (plastic and brittle) [3] has shown that forced longitudinal elastic oscillations in rod-like projectiles have low amplitude and high frequency and, thus, do not have a noticeable effect on final penetration depth.

In the framework of this study we use the developed model for the analysis of the influence of geometrical non-homogeneity and the size of projectile on penetration. The available experimental data by Forrestal et al. [4] and Warren [5] on penetration in grout and limestone were used to compare the observed results with calculations.

MODEL OF PENETRATION OF NON-DEFORMABLE PROJECTILE INTO SEMI-INFINITE TARGET

Consider the normal penetration of an axisymmetric rigid projectile into a semi-infinite target (see Fig.1).

In Fig. 1 we define the following notations and assumptions: σ_1 — lateral surface of projectile (i. e. projectile surface without its rear surface), $\sigma_2 = \{x = 0,\ r \geq d/2\}$ — target free surface, $\sigma_3 = \{0 \leq x \leq (P - L),\ r = d/2\}$ — free surface of entry hole. The direction of projectile motion coincides with axis x which is its axis of symmetry.

Main model parameters are: $u(t)$ — projectile velocity; L — projectile length; $P(t) = \int_0^t u(\tau)\,d\tau$ — penetration depth; d — projectile diameter; U_s — striking (impact) velocity, m — projectile mass.

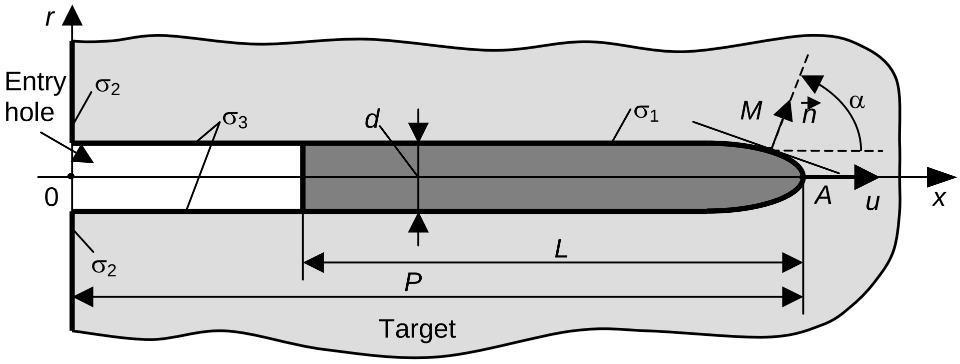


Figure 1. Normal penetration by a non-deformable projectile in a cylindrical coordinate system $0r\theta x$

The velocity field in the vicinity of projectile is approximated by the field of velocities $\vec{v}$ for irrotational motion of ideal incompressible fluid. Defining $\vec{v}$ we assume, that the free surface of target $\sigma_2 = \{x = 0,\ r \geq d/2\}$ remains flat during penetration, and the free surface of the entry hole σ_3 formed by the tail part of the projectile is cylindrical: $\sigma_3 = \{r = d/2,\ 0 \leq x \leq (P - L)\}$, (see Fig. 1).

It is well known [6] that the equations for non-stationary irrotational movement of an ideal incompressible fluid with density ρ in the case of potential flow have the following form:

$$\nabla^2\Phi = 0, \quad \text{(Laplace equation)} \tag{1}$$

$$\rho(\partial\Phi/\partial t) + \rho|\mathrm{grad}\Phi|^2/2 + p = f(t), \quad \text{(Lagrange-Cauchy integral)} \tag{2}$$

where $\Phi(x,r,t,P)$ – velocity potential (i.e. $\vec{v} = -\mathrm{grad}\Phi$), p – static pressure component, $f(t)$ – arbitrary function of time, independent from coordinates and identical over the whole volume of fluid. Function $f(t)$ is obtained from the boundary conditions. We assume that in (2) static component of pressure on projectile surface $p = R_t$ where for R_t we use the analytical solution of a problem of quasi-static expansion of spherical cavity in target material which takes into account elastic and plastic (or brittle) response of material.

For the potential Φ we have the following Neumann boundary conditions:

$$\partial\Phi/\partial n = -u(t)n_x \text{ at } M(r,x)\in\sigma_1,\ \partial\Phi/\partial n = 0 \text{ at } M(r,x)\in(\sigma_2\cup\sigma_3), \qquad (3)$$
$$\Phi = O(1/R)\to 0 \text{ when } R=\sqrt{r^2+x^2}\to\infty,$$

where σ_1, σ_2, σ_3 — surfaces as indicated earlier, n_x — cosine of angle α between axis x and the normal to projectile surface σ_1, (see Fig. 1). Condition (3) is a condition of impenetrability of target material across the projectile surface σ_1, cylindrical surface σ_3, and target surface σ_2.

From the linearity of boundary value problem of Neumann (1) and (3) it follows that the velocity potential $\Phi(x,r,t,P)$ may be presented as

$$\Phi(x,r,t,P) = u(t)\varphi(x,r,P). \qquad (4)$$

Here, the function $\varphi(x,r,P)$ is a solution of Laplace equation

$$\nabla^2\varphi = 0 \qquad (5)$$

with boundary conditions

$$\partial\varphi/\partial n = -n_x \text{ when } M(x,r)\in\sigma_1,\ \partial\varphi/\partial n=0 \text{ at } M(x,r)\in(\sigma_2\cup\sigma_3),$$
$$\varphi = O(1/R)\to 0 \text{ when } R\to\infty. \qquad (6)$$

Notice, that $\varphi(x,r,P)$ depends on P as on parameter, which, in its turn, depends on time t.

Thus, the velocity field in the proposed model is defined by the expression

$$\vec{v} = -u(t)\ \text{grad}\ \varphi(x, r, P). \qquad (7)$$

As in [7], we use the Lagrange-Cauchy equation (2) to develop the equation of motion for the penetrator. Substituting velocity potential (4) ($\Phi = u(t)\varphi(M,P)$, $\frac{\partial\Phi}{\partial t} = \varphi\frac{du}{dt} + \varphi'_P u^2$, $\text{grad}\Phi = u\,\text{grad}\varphi$) in the Lagrange-Cauchy integral (2) and considering this integral on the projectile surface, the left-hand part of (2) would define contact pressure $p_c(M,P)$ of target material on the projectile

$$p_c\left(M,P\right)=\rho\varphi\left(M,P\right)\frac{du}{dt}+\left(\left|\operatorname{grad}\varphi\left(M,P\right)\right|^2+2\varphi'_P\left(M,P\right)\right)\frac{\rho u^2}{2}+R_t\,. \tag{8}$$

A full decelerating force N acting on the projectile is obtained by integration over the contact surface of the projectile and target $\sigma(P)$ of the projection $(-p_c(M,P)\cdot n_x(M,P))$ of the contact pressure (8) onto the axis x. Here $n_x(M,P)$ is the cosine of an angle which is formed by a unit normal to the projectile surface at point M with the x axis (see Fig. 1).

Equating a projectile's resistance force N to its inertia force $m\dot{u}$, we have ballistic equation of motion for a non-deformable projectile:

$$m\dot{u}=N,\quad u(0)=U_0, \tag{9}$$

where $U_0 = U_s$ is the striking velocity.

Accounting for (8), equation (9) can be written as

$$m\dot{u}=-\rho A\left(P\right)\dot{u}-0.5B\left(P\right)\rho u^2-C\left(P\right)R_t\,,\ u(0)=U_0. \tag{10}$$

In (10) the coefficients A, B, C depend on projectile shape, penetration depth P and are defined by the formulas

$$A\left(P\right)=\int\limits_{\sigma(P)}\varphi\left(M,P\right)n_x\left(M,P\right)d\sigma_M\,,\ C\left(P\right)=\int\limits_{\sigma(P)}n_x\left(M,P\right)d\sigma_M\,,$$
$$B\left(P\right)=\int\limits_{\sigma(P)}\left(\left|\operatorname{grad}\varphi\left(M,P\right)\right|^2+2\varphi'_P\left(M,P\right)\right)n_x\left(M,P\right)d\sigma_M\,. \tag{11}$$

A number of expressions exists for the value of static penetration resistance of target material R_t depending on material rheology. For elastic-ideally plastic and elastic-brittle materials see, for example, [1,2,8-10], and for post-yield strain-hardening material — [11].

For penetration into limestone, Frew et al. [12] established that penetration resistance depends on penetrator dimensions:

$$R_t = \Psi + \phi\,(a_0/a), \tag{12}$$

where Ψ = 607 MPa, ϕ = 86 MPa, 2 a_0 = 25.4 mm and a is the projectile radius (mm).

To find velocity potential φ from (5), (6), the indirect method of boundary equations [13] was used, where φ is represented by a simple layer potential with source density distributed on the projectile and entry hole surfaces.

AN ANALYSIS OF ELASTIC OSCILLATIONS IN NON-HOMOGENEOUS PROJECTILE

While evaluating the influence of elastic longitudinal oscillations in a projectile during its penetration, we assume that the projectile has the shape of circular cylinder with a diameter d and length $L=L_1+L_2$ (Fig. 2).

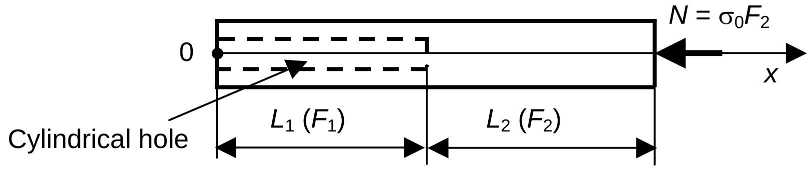


Figure 2. Scheme of non-homogeneous projectile. F_1, F_2 — cross section area.

These oscillations are considered separately from projectile motion as a rigid body and the estimation of their influence on the transportation velocity of rigid penetrator is made. A penetration of long rigid projectile occurs at almost constant acceleration $\dot{u}$ (as solution of (10) shows) and, thus, at almost constant braking force $N=-m\dot{u}=\sigma_0 F_2>0$, acting on projectile (here u — solution of (10)). Besides, the value of force N changes only weakly at small variations of initial condition $u(0)=U_0$ in (10).

Thus, to evaluate the influence of forced elastic oscillations in the projectile we use a solution of a problem of longitudinal oscillations in a rod similar to, e.g., Timoshenko [14] with one free end ($x=0$), and compressive stresses ($-\sigma_0$) instantly applied to another ($x=L$) free end (at the moment of time $t=0$):

$$\frac{\partial}{\partial x}\left(k(x)\frac{\partial w}{\partial x}\right)-\rho(x)\frac{\partial^2 w}{\partial t^2}=-\sigma_0\delta(x-L)H(t),$$
$$k(x)=\begin{cases}\alpha^2 E_p, & 0\le x\le L_1\\ E_p, & L_1<x\le L\end{cases},\quad \rho(x)=\begin{cases}\alpha^2\rho_p, & 0\le x\le L_1\\ \rho_p, & L_1<x\le L\end{cases},\ \frac{F_1}{F_2}=\alpha^2, \tag{13}$$

where $w(x,t)$ — displacement of cross-section x of a rod at time t, E_p, ρ_p — Young modulus and density of penetrator, H, δ — Heaviside and Dirac functions.
Initial and boundary conditions for (13)

$$w(x,0)=0,\ \frac{\partial w}{\partial t}(x,0)=0\,. \tag{14}$$

$$\frac{\partial w}{\partial x}(0,t)=\frac{\partial w}{\partial x}(L,t)=0, w(L_1-0,t)=w(L_1+0,t), \alpha^2\frac{\partial w}{\partial x}(L_1-0,t)=\frac{\partial w}{\partial x}(L_1+0,t)\,. \tag{15}$$

Solution of problem (13)-(15) is

$$w(x,t) = w_r + w_{osc} = -\frac{\sigma_0\delta_0 t^2}{2} - \sigma_0 \sum_{i=1}^{\infty} \frac{\delta_i}{p_i^2}(1-\cos p_i t)V_i(x),\ 0 \le x \le L\,, \quad (16)$$

where $V_i(x) = \begin{cases} \cos(p_i x/c_0),\ 0 \le x \le L_1, \\ \cos(p_i L_1/c_0)\dfrac{\cos(p_i x/c_0) + \tan(p_i L/c_0)\sin(p_i x/c_0)}{\cos(p_i L_1/c_0) + \tan(p_i L/c_0)\sin(p_i L_1/c_0)},\ L_1 \le x \le L, \end{cases}$

$c_0 = \sqrt{E_p/\rho_p}$ — sound velocity; p_i, $i = 0,1,2,\ldots$ — solutions of frequency equation

$$\cos(\gamma_i\beta)\sin[\gamma_i(1-\beta)] + \alpha^2\sin(\gamma_i\beta)\cos[\gamma_i(1-\beta)] = 0\,,\ \gamma_i = p_i L/c_0\,, \beta = L_1/L\,, \quad (17)$$

$\delta_i = V_i(L)/(V_i,V_i)$, $(V_i,V_i) = \int_0^L \rho(x)V_i^2(x)\,dx$, and $w_r = -\sigma_0\delta_0 t^2/2$.

Initial velocity of penetration of deformable projectile U_0 in target accounting for an elastic response of the penetrator is

$$U_0 = U_s - \sigma_0 c_0/E_p\,, \quad (18)$$

where U_s — projectile velocity before its meeting with target, i.e. impact velocity. For a non-deformable projectile $U_0 = U_s$. Thus, actual penetration velocity $u(t)$ and penetration depth $P(t)$ are determined by formulas

$$u(t) = \bar{u}(t) + \dot{w}_{osc}(L,t),\ P(t) = \int_0^t u(\tau)\,d\tau\,, \quad (19)$$

where $\bar{u}$ — velocity of rigid projectile found from (10).

NUMERICAL IMPLEMENTATION

Calculations were carried out for steel projectile impacting grout targets [4]. For grout target $R_t = Sf_c$, $S = 72.0 f_c^{-0.5}$ [15]. Projectile – length 88.9 mm, outer diameter 12.92 mm, inner diameter 6.35 mm, CRH 4.25 [4]. Homogeneous and non-homogeneous projectiles have same mass and head shape.

Integro-differential equation (10) was solved numerically. Fig. 3 shows the velocity of penetration. Figure 4 compares $\dot{w}_{osc}(L,t)$ component of (19) for homogeneous and non-homogeneous projectiles for a fragment of Fig.3. Figure 5 gives a comparison of final penetration depth with experimental data.

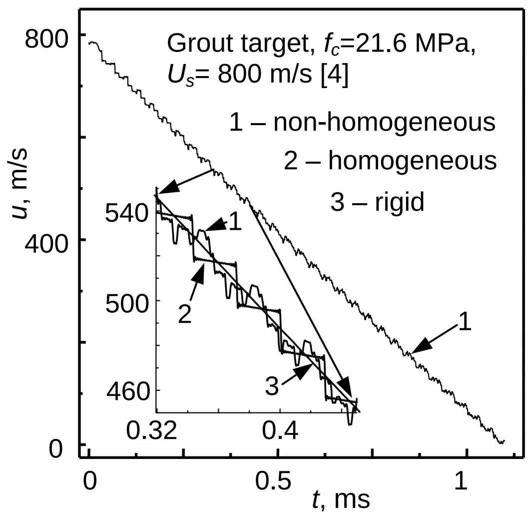


Figure 3. Penetration velocity (19) taking into account oscillations of the projectile.

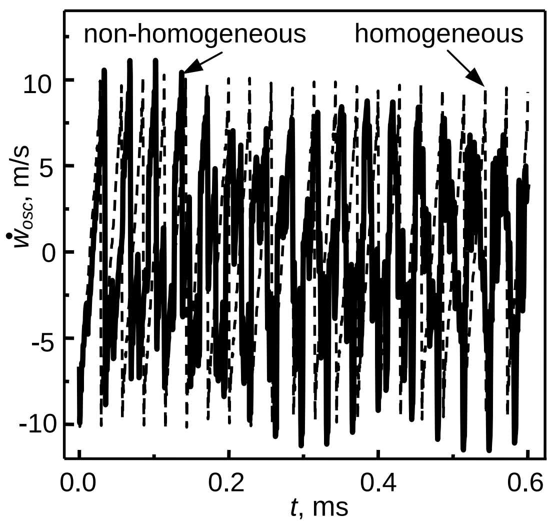
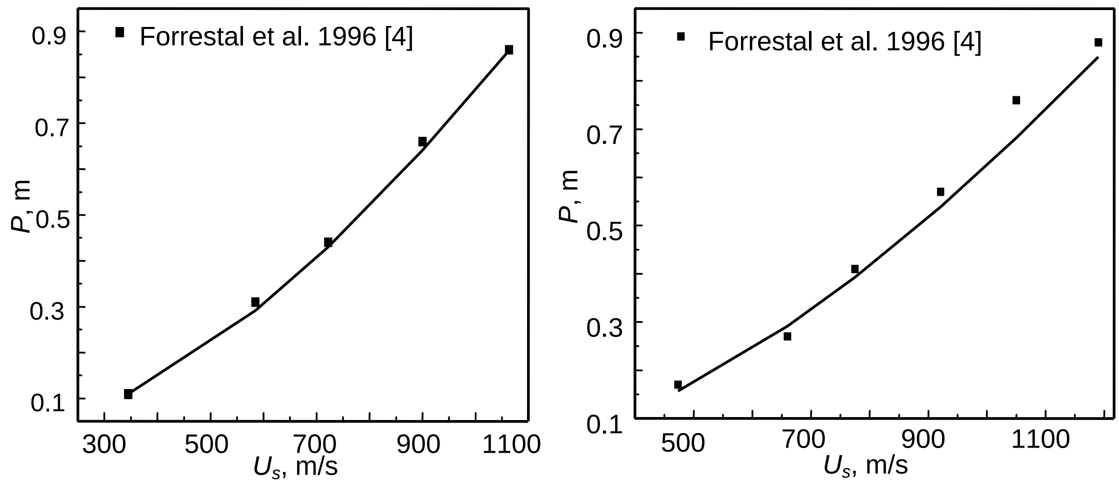


Figure 4. Oscillation velocity.



(a) (b)

Figure 5. Comparison of experimental and calculated dependence of penetration depth for grout targets with unconfined compressive strength 13.5 MPa (a) and 21.6 MPa (b).

CONCLUSION

Comparison of the modeling results with experimental data for grout and limestone targets [4,12] show good agreement without additional fitting parameters.

The calculations show that oscillations of the tip of non-homogeneous projectile are more complex then for homogeneous one. Amplitudes of velocities and displacements are varying around values for homogeneous projectile:

$$\dot{w}_{osc}^{max} \approx c_0 \frac{\sigma_0}{E_p}, \quad w_{osc}^{max} \approx L \frac{\sigma_0}{2E_p}.$$

For targets with lower strength these amplitudes are smaller because $\sigma_0 \sim R_t <<$ projectile yield limit. Besides, these amplitudes decrease with increase in projectile diameter (inverse quadratic dependence).

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