

## PENETRATION MODEL TAKING IN MIND VISCOSITY PROPERTIES OF THE IMPACTED BODY MATERIALS

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A developed model for penetration of a rod into a semi-infinite plate within the impact velocity range of 1000...4000 m/s is presented. The model is based on the hydrodynamic approach with its further modification by considering viscous properties of materials of impacting bodies along with their strength properties. A form of the viscosity member in the balance and motion equations associated with kinematics characteristics of the penetration and geometric parameters of penetrators is suggested on the basis of generalization of modern information on metal viscosity coefficients under high deformation rates. A good correspondence of calculated dependencies with experimental data on the penetration of the penetrator into the target is shown.

### INTRODUCTION

Consideration of influence rheological properties of materials on high-rate deformation is important when studying and simulating different phenomena in mining, explosion physics, and when studying impact deforming solids. The paper dwells upon developing of a model of impact penetration of a metal cylindrical penetrator into a semi-infinite target.

Papers [1, 2] propose a modified hydrodynamic model for penetration within the velocity range below 3000 m/s, taking into account the strength properties of the penetrator and target. The consideration of this factor allowed obtaining more reliable assessments of penetration conditions, however one failed to achieve a good agreement with test results and some test data remained unexplained. We think one of the reasons is a lack of consideration of as important property of materials as viscosity.

In order to substantiate the idea on the necessity to take into account viscosity of metals when developing the model for penetration, the values of  $\mu$  were calculated according to different methods. Table 1 presents the range of extreme values of this coefficient for the specific metal, as well as the level of deformation rates  $\dot{\epsilon}$ , at which

the coefficient was obtained in tests. One can see the values of absolute viscosity of the same metal, determined by different authors by principally different methods, differ no more than in order of magnitude, that is quite sufficient to assess the level of forces of viscous resistance against deforming at impact of solids.

In order to assess the deformation rate  $\dot{\epsilon}$  when the penetrator penetrates the target one uses the following formula

$$\dot{\epsilon} = V_0/d, \quad (1)$$

where  $d$  is the diameter or parameter, characterizing the cross section area (middle) of the penetrator. One considers  $d$  for different penetrators is within the range from 1 to 100 mm. Therefore the limits of change of  $\dot{\epsilon}$  in the range 1000...3000 m/s is  $10^4 \dots 3 \cdot 10^6$  1/s, i.e. just the same range, where the data of Table 1 were obtained.

Then to assess the level of viscous component of resistance against deforming at impact  $\sigma_v$ , one uses:

$$\sigma_v = \mu \dot{\epsilon} = \mu V_0/d \quad (2)$$

Table 1  
Coefficients of dynamic viscosity of metals

Coefficients	Duralumin	Steel	Copper	Titanium
$\mu \cdot 10^{-5}$ , P	0.2...1.0 (0.6)	0.2...1.8 (1.0)	2.0...2.7 (2.4)	4.2...4.4 (4.3)
$\dot{\epsilon}$ , 1/s	$5 \cdot 10^3 \dots 10^7$	$5 \cdot 10^3 \dots 10^7$	$5 \cdot 10^3 \dots 10^7$	-

Comment: Value in brackets is average  $\mu$ .

Figure 1 shows the results of comparative analysis of inertia, strength and viscous resistance against deforming of metals at impact. One can see in the velocity  $V_0$  range of 800... 2500 m/s the resistance, proportional to the velocity, is comparable in magnitude to inertial and strength ones, and can exceed the last one, which doesn't considerably depend on the impact velocity.

## PHYSICAL-MATHMATICAL MODEL

In order to develop a physical model for penetration of a penetrator one uses a modified hydrodynamic model [1, 2] with the following modification by taking into account the viscous properties of materials of impacting solids [3]. One considers penetration of a cylindrical penetrator of  $L$  length and  $d$  diameter into a target (plate).

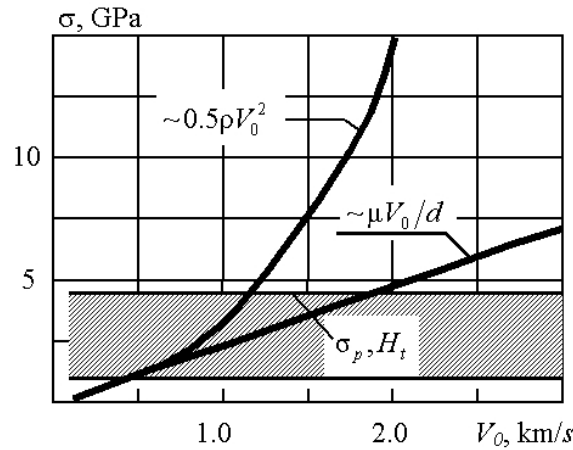


Figure 1. Comparative analysis of inertial, viscous and plastic resistance against deforming of steel at impact and  $\mu=1 \cdot 10^5$  P,  $d=4$  mm

The penetration velocity or the velocity of the contact interface “penetrator–target” is denoted  $U$ , the velocity of the back end of the penetrator regarding the moving contact surface is denoted  $(V-U)$ . These velocities are characteristic values of motion, defining, in particular, the level of inertial forces, appearing in the penetrator  $k_p \rho_p (V-U)^2$  and target  $k_t \rho_t U^2$ .  $H$  is the depth of penetration of the penetrator into the target. Our model uses the yield stress  $\sigma_p$  for the penetrator and dynamic hardness  $H_t$  of target material as the aforementioned values [1, 4].

The value for the viscous component of resistance against deforming can be written as  $k_p \mu_p \dot{\epsilon}_p$  and  $k_t \mu_t \dot{\epsilon}_t$  for the penetrator and target, respectively, having introduced into the formula the coefficients  $k_p=k_t=1/2$  to describe the shape of the deformed region in the penetrator and target, as well as characteristic deformation rates for the penetrator  $\dot{\epsilon}_p$  and target  $\dot{\epsilon}_t$ .

The stress state at the critical point of the deformed material of the penetrator and target is defined by the sum of the inertial, viscous and strength components. Having equaled pressures at both sides of the contact surface “penetrator–target” at some moment after penetration, one obtains the following equation (at the point of contact at the axis of symmetry of the interface):

$$\frac{1}{2} \rho_p (V-U)^2 + \frac{1}{2} \mu_p \dot{\epsilon}_p + \sigma_p = \frac{1}{2} \rho_t U^2 + \frac{1}{2} \mu_t \dot{\epsilon}_t + H_t. \quad (3)$$

Considering the penetration of the deforming penetrator, one naturally distinguishes the characteristic deformation rates of the penetrator  $\dot{\varepsilon}_p \sim (V - U)$  and target  $\dot{\varepsilon}_t \sim U$ .

The change of intensity of target deformation rate  $\dot{\varepsilon}_t$  from great values at the contact surface to practically zero takes place in a very thin layer of the target, which thickness is comparable with the penetrator diameter. The width of the region of structural conversions in the target is  $(0.5 \dots 0.7)d$  along the side surface of the crater. In direction to the crater bottom the value increases a bit, achieving the biggest size  $(0.9 \dots 1.5)d$  just near under its bottom.

Thus, when composing the balance of forces at an arbitrary moment one can consider the diameter of the cross section of the penetrator as the characteristic linear size of the region of the deformation flow. The size defines the region of the metal, which makes the most contribution to the resistance against penetration, as the very layer consumes the most energy for the viscous-plastic deformation of the target, and, which is the most important, there is the largest gradient of the flow characteristics.

According to the aforementioned for the first approximation with the accuracy to the constant multiplier in order to find the deformation rate of the penetrator and target one can use:

$$\dot{\varepsilon}_p = (V - U)/d; \quad \dot{\varepsilon}_t = U/d. \quad (4)$$

Including (4) into (3), one has:

$$\frac{1}{2}\rho_p(V - U)^2 + \frac{1}{2}\frac{\mu_p}{d}(V - U) + \sigma_p = \frac{1}{2}\rho_t U^2 + \frac{1}{2}\mu_t \frac{U}{d} + H_t, \quad (5)$$

from which:

$$U = \frac{1}{1 - \frac{\rho_t}{\rho_p}} \left( V + \frac{\mu_p(1 + \frac{\mu_p}{\mu_t})}{2\rho_p d} - \sqrt{\frac{\rho_t}{\rho_p} V^2 + \frac{\mu_p(\frac{\mu_p}{\mu_t} + \frac{\rho_t}{\rho_p})}{\rho_t \rho_p} V + \frac{\mu_p^2(1 + \frac{\mu_p}{\mu_t})^2}{4\rho_p^2 d^2} + 2(H_t - \sigma_p)(1 - \frac{\rho_t}{\rho_p})/\rho_p} \right) \quad (6)$$

The rate of “working-off” or decrease of the penetrator length:

$$dl/dt = -(V-U). \tag{7}$$

The rate of penetration (formation of crater):

$$dH/dt = U. \tag{8}$$

The equation for motion of the deformed penetrator into the target is:

$$\rho_p l dV/dt = -\sigma_p - \frac{1}{2} \mu_p \dot{\epsilon}. \tag{9}$$

In our case the total penetration time is proportional to the penetrator length and just isn't proportional on the diameter. In turn the stress state, configuration and sizes of the deformation region in the penetrator and target, defining the balance of forces on the contact interface, are not connected with the penetrator length. Thus in order to define the flow regime one should introduce at least two characteristic linear sizes: longitudinal and transversal into the model. They are independent for the first approximation and are directly connected with the two main geometrical parameters of the penetrator: its length and diameter. For the features of the process, which is reflected in equation (9), the value  $\dot{\epsilon}$  according to the aforementioned idea and condition (7) is:

$$\dot{\epsilon} = \frac{1}{L} \cdot \frac{d(l-L)}{dt} = \frac{1}{L} (V-U), \tag{10}$$

where the penetrator length  $L$  is the characteristic parameter.

Then equation (9) is

$$\rho_p l dV/dt = -\sigma_p - \frac{1}{2} \mu_p (V-U)/L \tag{11}$$

The solution of system of equations (5)...(11) depending on the ratio  $\sigma_p/H_t$  can be as the following cases

Case I:  $\sigma_p/H_t < 1$ .

In this case the penetrator behaves as a viscous-plastic matter, shortening and braking to the end of penetration. When the pressure at the surface of contact is decreased to  $H_t$ , the velocity of penetration is zero. This takes place at the back end velocity equal to  $V_1$

$$V_1 = \sqrt{2(H_t - \sigma_p)/\rho_p + \mu_p^2/4\rho_p^2 d^2} - \mu_p/2\rho_p d. \tag{12}$$

At  $V \leq V_1$  the formation of the crater ends and the target can be considered as a rigid solid. The penetrator continues shortening, and brakes without penetration into the target. If  $\sigma_p = 0$ ;  $\mu_p \neq 0$ , this corresponds to the penetration of the penetrator of liquid, perfectly viscous material (for example, melted metal). The beginning of penetration of such penetrator corresponds to the impact velocity, determined from (12) at  $\sigma_p = 0$ .

At  $V_0 > V_1$ , the liquid penetrator can penetrate into the target and brake only due to viscous resistance.

Case II:  $\sigma_p/H_t > 1$ .

If at penetration due to braking the penetrator velocity is decreased in such manner, that the pressure at the contact surface is less than  $\sigma_p$ , the deformation stops, and the following penetration corresponds to the motion of a non-deformable solid in a viscous-plastic matter. This takes place at  $U = V = V_2$ :

$$V_2 = \sqrt{2(\sigma_p - H_t)/\rho_t + \mu_t^2/4\rho_t^2 d^2} - \mu_t/2\rho_t d \quad (13)$$

At  $V_0 > V_2$ , the total depth of penetration is determined by the sum of penetration depth in hydrodynamic mode  $H_h$  and penetration of the penetrator as a rigid body  $H_l$ .

Case III:  $H_t = 0$ ,  $\mu_t \neq 0$  corresponds to the penetration into the target of perfectly viscous matter, for example, into a target with temperature, close to the melting point

Using numerical methods the different variants of the problem of penetration have been analyzed. Some results are presented in Figure 2.

Thus, for variant *a* the ratio of main parameters was  $\rho_p/\rho_t = 1$ ;  $\mu_p/\mu_t = 1$ , i.e. the same density and viscosity of the penetrator and target, then  $\sigma_p/H_t < 1$ , as for most constructional materials. For variant *b* the density, viscosity and strength of the penetrator exceed the corresponding characteristics of the target. It's clear as the  $t$  increases, the penetration depth  $H$ , first, increases quickly, then the intensity of its increase diminishes. The period of fast increase is 75...90% of the total interaction time and characterized by just constant value of the derivative  $dH/dt$ . As soon as in variant *a* the velocity of the back end of the penetrator becomes lower than the value  $V_1$ , the crater stops deepening, but the consumption ("working-out") continues. At  $V_0$ , equal to 1000 m/s, the additional shortening of the penetrator achieves 2...4% of the initial length and changes little as  $V_0$  grows. In variant *b* at  $V \leq V_2$  the penetrator stops deforming, but the depth of the crater continues increasing.

As the penetrator length increases the crater depth increases also, especially in the range  $>2000$  m/s, where  $H$  becomes proportional to  $L$ . The change of  $d$  results in the change of the comparative role of the viscous resistance in the general balance of forces.

One considers the range of application of the developed physical model of penetration. The supposition on non-compressibility limits the upper limit of  $V_0$  by the

value 3000...3500 m/s. The permissible lower limit of the range application regarding  $V_0$  is defined by the value  $V_2$ .

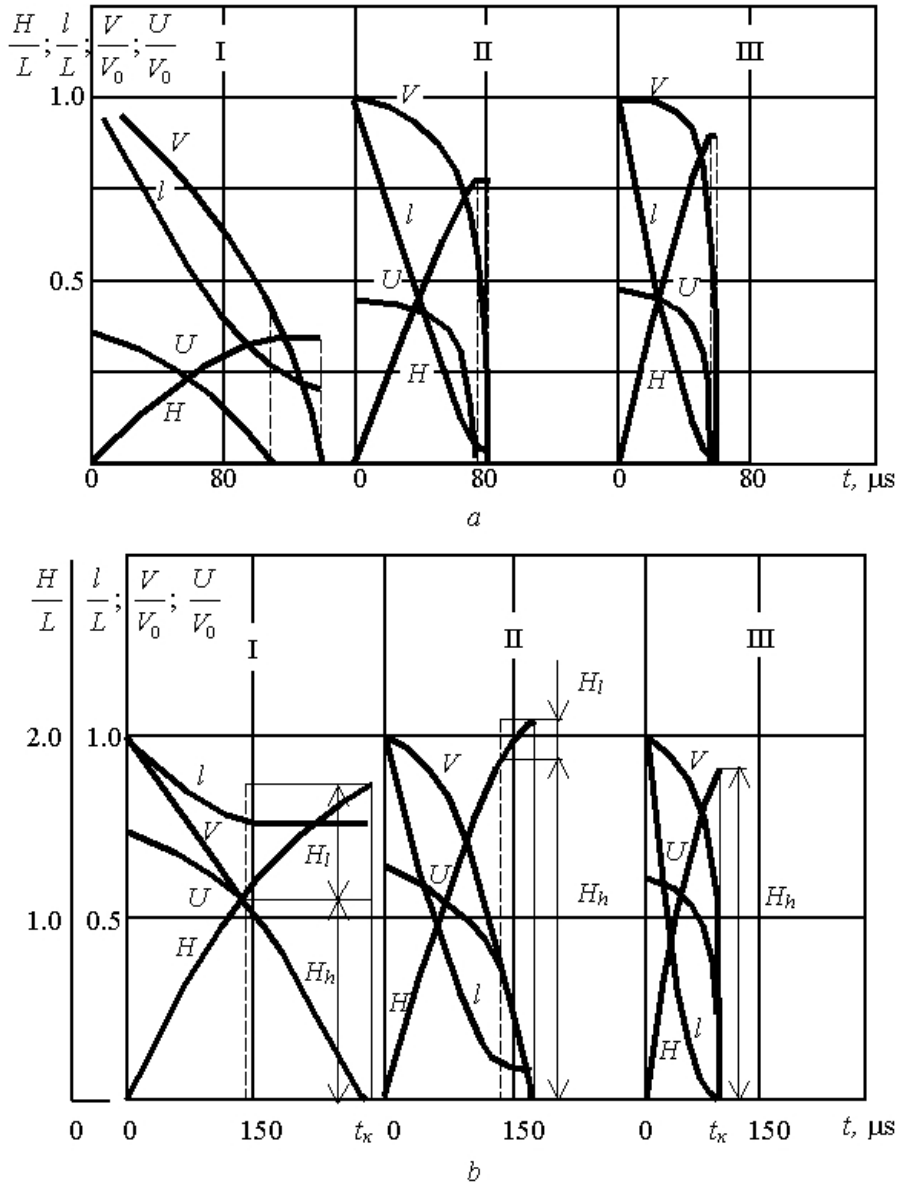


Figure 2. Penetration parameters versus time  $t$  during interaction at  $L/d=10$ ; a -  $\rho_p/\rho_r=1$ ,  $\mu_p/\mu_r=1$ ,  $\sigma_p/H_r=0.5$ ; b -  $\rho_p/\rho_r=2.85$ ,  $\mu_p/\mu_r=1.6$ ,  $\sigma_p/H_r=1.8$ ; I -  $V_0=1000$  m/s; II -  $V_0=2000$  m/s; III -  $V_0=3000$  m/s.

The comparison of the calculation results with test data on penetration of the penetrator in the semi-infinite target shows, the model is qualitatively correct in

describing the process. Model pairs were considered, which corresponded to the calculation variants  $a$  and  $b$ .

The results of comparative analysis show the calculated lines are close enough to the test data. The best agreement is seen in the range of velocities above 1200 m/s. At  $V_0 < 1200$  m/s the calculated values of  $H$  are higher, and at  $V_0 > 2000$  m/s, vice versa, some lower in comparison with the test data.

The comparison with the test shows in a number of cases the best agreement is observed if the values  $\mu$  are selected as different, depending on the range of  $\dot{\epsilon}$ . The comparison also shows the increase of  $\dot{\epsilon}$  causes the decreases of  $\mu$ .

The specification of the rheological properties of materials, the introduction of function  $\mu(\dot{\epsilon})$  into the system of the determinant equations allows to improve the presented model.

## CONCLUSION

1. The physical model of penetration of the penetrator into the metal target, taking into account viscosity, as well as inertia and plastic properties of materials of impacting solids.

2. The inclusion of viscosity allows reflecting the main features of the impact deformation on the base of modern suppositions on rheological properties of metals at high-rate deformation more completely, that is confirmed by tests.

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