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CONSTITUTIVE MODEL FOR BOROSILICATE GLASS AND APPLICATION TO LONG-ROD PENETRATION

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Numerical and analytical simulations of projectiles penetrating brittle materials like ceramics and glasses are a very challenging problem. The difficulty comes from the fact that the yield surface of brittle materials is not well characterized (or even defined) and the failure process might be changing material properties. Penetration tests on glass and silicon carbide found in the literature show that the target fails ahead of the projectile (the ``failure wave'' is faster than the penetration velocity). So, during penetration, the projectile ``sees'' failed glass ahead and, consequently, numerical simulations should also use failed material properties to give accurate results. This paper shows how, with a test explained elsewhere and consisting of compression under confinement of cylindrical samples and the help of an analytical model, it is possible to determine the elastic and the Drucker-Prager constants for damaged borosilicate glass (or, in principle, any other brittle material). The constants found with this procedure were used afterwards as the material properties of the glass in numerical simulations of a gold rod penetrating intact borosilicate glass at impact velocities ranging from 0.5 to 3 km/s. As shown in the paper the numerical simulations match the experimental penetration velocity.

INTRODUCTION

Rasorenov et al. [1] introduced the concept of failure waves traveling in glass. This lead to a fair amount of work by other authors, for example, Bourne ([2],[3]), Partom [4] and many more. Recent experimental work by Orphal et al. [5], on long rods impacting glass targets, shows that borosilicate and lead glass fail ahead of the projectile for impact velocities ranging from 0.5 to 3 km/s. All the above references indicate that during the penetration process the projectile is in contact with failed glass and not with intact glass material, even for impact velocities on the order of the speed of sound of the glass.

Consequently, to achieve realistic numerical simulations of penetration into glass, characterization of the target cannot be limited to the intact material but should also include failed material as well. Unfortunately, the material models found in the literature are still semi-empirical. For example, Holmquist [6] presented an indirect way to characterize glass by using static test data, flyer plate impacts and ballistic results and iterating on the unknown constants (damage constants, fractured strength, etc.) until computational results match the experiments.

This paper presents the constitutive model, found with a technique presented elsewhere [8], consisting of confined compression tests on damaged glass, assuming the plasticity model is Drucker-Prager [7]. The constitutive model was implemented on CTH to predict the penetration velocities of gold rods into intact borosilicate glass.

MATERIALS

The characterization tests were performed on intact and pre-damaged samples of borosilicate glass. The brand name is Borofloat® 33, manufactured by Schott Glass using a float process. The ultrasound mechanical properties of the intact glass were provided by [9] and are shown in **Table 1**.

The sleeves used in the compression under confinement tests were made of Vascomax C350 maraging steel. Its density, elastic modulus and Poisson's ratio are, respectively, 7.83 g/cm^3, 205 GPa and 0.28. The yield strength is around 2.5 GPa, purposely high to avoid yielding during the tests. Tapered Tungsten Carbide anvils were used to load the sample in compression. The Tungsten Carbide material has a nominal elastic modulus of 627 GPa and a compression strength of 5 GPa.

For the penetration experiments the projectile material was gold with a density of 19.2 g/cm^3. The strength of the projectile is assumed to be negligible.

EXPERIMENTAL TECHNIQUES

Material Characterization under confinement pressure

The test used to characterize the glass under confinement consists of a cylindrical specimen with a radius of 3.175 mm and length of 12 mm. The specimen is inserted in a Vascomax steel sleeve and compressed in the z direction from the free ends. The sleeve outer diameter is 12.70 mm. The specimen was, on some of the tests, pre-damaged, i.e. it was exposed to two cycles of intense heat (500 C) for twenty minutes and afterwards suddenly plunged into ice water. With this procedure a few cracks were produced in the specimen but, in general, the specimen did not crumble and could be manipulated without being destroyed.

An axial compressive stress is applied to the specimen with an MTS servohydraulic machine by means of two tungsten carbide platens (silicon carbide was also used). The variables recorded during the test are: axial stress in the specimen, measured by a load cell in the MTS machine; axial strain in the specimen, measured by a clip gage placed on the top and bottom platens; axial and hoop strain in the sleeve, measured respectively by a vertical and annular strain gage on the sleeve. Some of the tests were tested with load increasing monotonically until failure and some others consisted of 5-10 load-unload cycles. More details about this experimental technique are available in Dannemann [10].

Long Rod Penetration Experiments

Reverse ballistics experiments were performed at Ernst Mach Institute. The experimental set-up and main results are described in detail by Orphal et al. [5]. The target was made of the same borosilicate glass used in the characterization experiments. The projectile was a gold rod, 70 mm long and 1 mm diameter. The penetration process was recorded with the help of four X-Ray cameras. Position of the rod vs. time was obtained for different impact velocities allowing the penetration velocity of the rods into the glass to be determined. The penetration velocity during the quasi-steady state of penetration, where it stays approximately constant, is used to check the material model for the glass.

RESULTS FROM CHARACTERIZATION: CONSTITUTIVE MODEL

From the confined compression tests of the glass specimens it is possible [14] to obtain the Equivalent Stress vs. Pressure behavior. This was done for both the intact and pre-damaged borosilicate glass. A Drucker-Prager [7] plasticity model was assumed for the glass ($Y = Y_0 + \beta P$). The strength was limited by a cap $(\overline{Y})$. The slope was found to be β=1.8. The strength at zero pressure, Y_0, was estimated by doing three compression tests of the damaged specimen without confinement. Those results imply that Y_0 is approximately 40 MPa.

The average values and standard deviation are shown in **Table 1**. Intact and ultrasound measurements are also provided for comparison. The modulus changes very little from intact to pre-damaged (thermally shocked) glass. Even when the glass is severely damaged, as for example in a multiple cycle load-unload test, the average modulus decreases less than 10% when compared to the modulus of the material determined from MTS data.

Table 1: Average material properties inferred from the tests for predamaged glass. Intact properties are provided for comparison

	E (GPa)	ν	β	$\overline{Y}$(GPa)	Y_0(MPa)
Damaged (MTS)	56 ± 5	0.20 ± 0.04	1.8 ± 0.2	1.8 ± 0.2	43 ± 13
Predamaged (ultrasonic)	60 ± 2	0.18 ± 0.02	-	-	
Intact (MTS)	59 ± 2	0.18 ± 0.01	-	-	
Intact (ultrasonic)	62.2 ± 0.1	0.200 ± 0.007	-	-	

Figure 1 shows, as an example, in graphical form the elastic and shear moduli calculated for all the tests and cycles. There is no clear trend to indicate that the modulus increases or decreases with load cycling, instead it seems that the modulus stays the same throughout the test. Similar plots for the Poisson's ratio and the slope of the Drucker-Prager also show little change in the properties throughout the cycling (i.e. damaging) process.

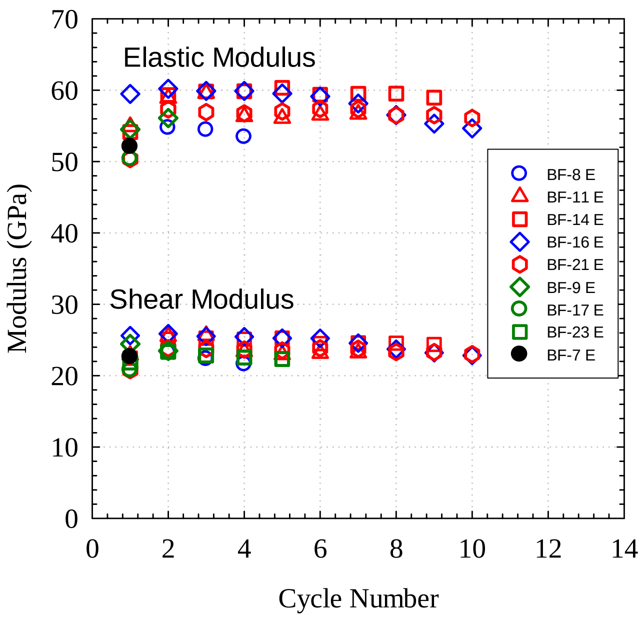

Figure 1: Elastic and shear modulus evolution for different tests and cycles.

The Drucker-Prager model for damaged borosilicate can, in summary, be written as:

$$Y = 0.043 + 1.8P \quad (P \ \ in\ GPa)\ ;\ \overline{Y} = 1.8\ GPa \qquad (1)$$

APPLICATION OF CONSTITUTIVE CONSTANTS: PENETRATION VELOCITY OF A GOLD ROD IN BOROSILICATE GLASS

CTH simulations

CTH simulations were 2-D axisymmetric with 10 cells across the radius of the projectile. The material properties used for the ``nominal" simulation are shown in Table 2. ρ is the density of the intact glass, C_s is the bulk sound speed, s the slope of the shock velocity vs. particle velocity curve and Γ the Gruneisen coefficient.

Table 2: Material properties used in the ``nominal'' CTH simulation

	ρ(g/cc)	C_s(km/s)	s	Γ	Y_0(MPa)	β	$\overline{Y}$(GPa)
Borosilicate Glass	2.5	3.86	0.	1.	50	1.8	1.8

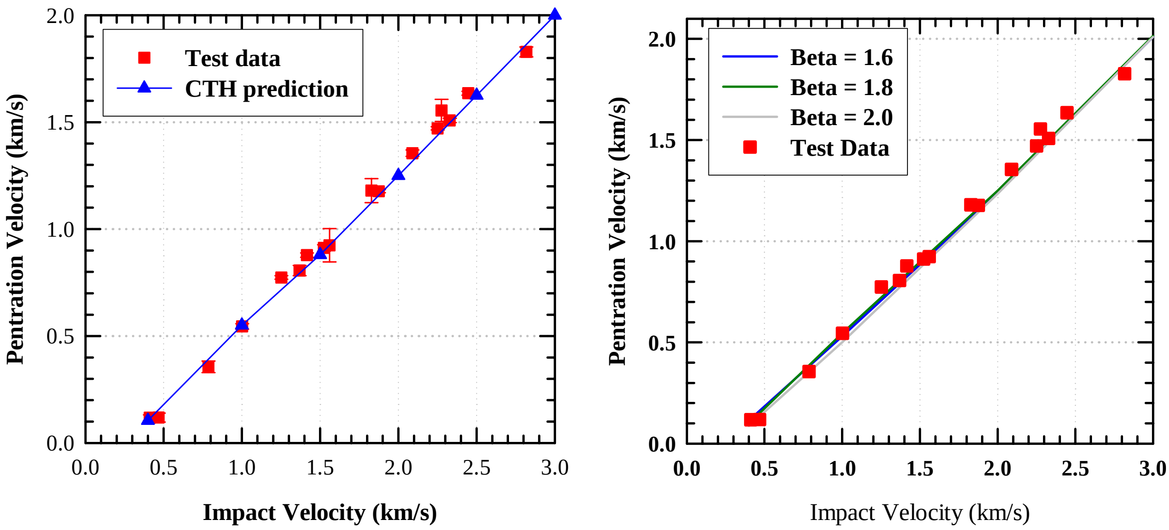

Figure 2: a) CTH results compared with the experiments, b) Sensitivity study with the β parameter

The values of s and Γ are an educated guess, but should not greatly influence the results of the computations. The value of β is the average of the slopes measured with the experimental technique described above.

Figure 2a) compares the penetration velocity from the computations with those obtained experimentally using the x-ray data. The agreement is excellent for the whole range of velocities tested.

Sensitivity study

A sensitivity study was performed to gain understanding of the influence of the different parameters ($\beta, Y_0, \overline{Y}$) on the penetration velocity of the projectile. The first series of simulations used the slope of the Drucker-Prager equation as a parameter. The β values were varied (1.6, 1.8, 2.0) keeping Y_0 and $\overline{Y}$ constant (50 MPa and 1.8 GPa, respectively). Figure 2b) clearly shows little sensitivity on the slope parameter in this range of impact velocities. The second round of simulations, see Figure 3a), kept β and $\overline{Y}$ constant while varying Y_0 from 25 to 200 MPa. As expected, at low velocities Y_0 does have an important effect on the results since a small strength in tension means less confinement of the glass. The third exercise used $\overline{Y}$ as the variable parameter (1.5, 2.0, 2.5 GPa); and the other two variables were held constant. The cap $\overline{Y}$, only comes into play at high pressures, i.e. high velocities. Even at high velocities Figure 3b) shows that its influence is rather small.

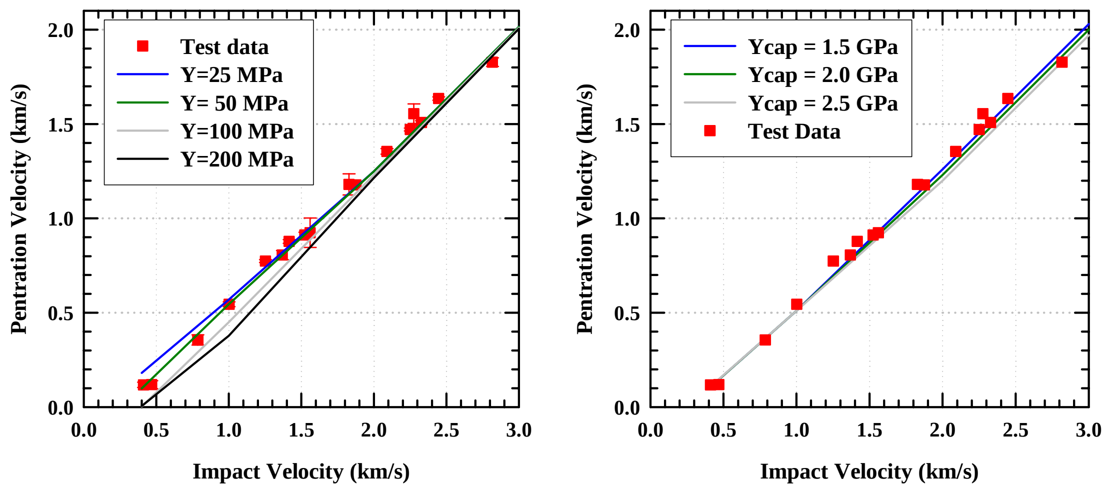

Figure 3: a) Sensitivity study with Y_0 b) Sensitivity study with $\overline{Y}$

A similar Y_0 sensitivity study was performed with the analytical model developed by Walker [17] to check that the high sensitivity at low impact velocities is not due to numerical artifacts. Figure 4 compares the analytical and numerical results for the two extreme cases studied, $Y_0 = 25\text{MPa}$ and $Y_0 = 200\text{MPa}$. The numerical and analytical match at low impact velocities is excellent, both showing the same sensitivity to Y_0.

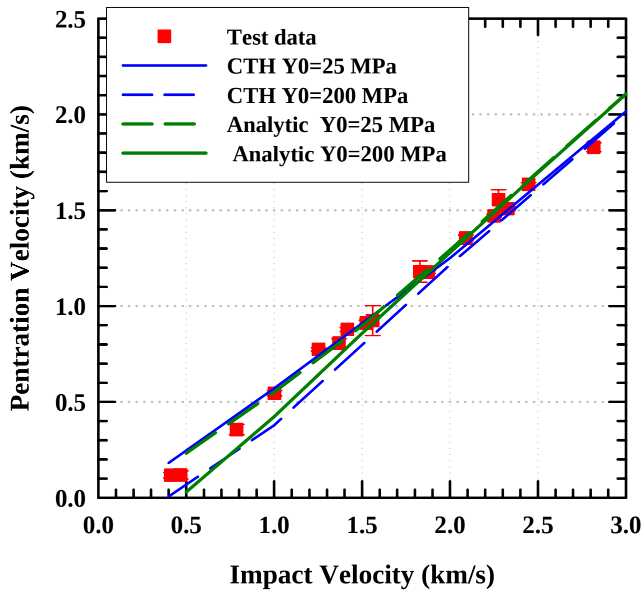

Figure 4: The sensitivity study was also performed with the analytical model by Walker [17] to confirm that it was not a numerical artifact

Clearly the single parameter that most affects the results at low impact velocities is Y_0. In contrast small variations in the slope of the Drucker-Prager , β, and the cap, $\overline{Y}$, do not affect the steady state penetration velocity. For computational purposes, an error of 10 or 20% in β or $\overline{Y}$ would not have a great impact on the prediction.

Conclusions

This paper has shown that it is possible to directly measure in the laboratory strength parameters of pre-damaged brittle materials that are relevant for penetration computations. The test used to characterize the material (borosilicate glass) consisted on the compression of a specimen confined by a metallic sleeve. By measuring the hoop strain of the sleeve the pressure in the specimen was derived and the constitutive (assumed to be Drucker-Prager with a cap) model inferred.

The constants found in the model were used in CTH computations to predict the penetration velocity of gold rods into intact glass. Comparisons are performed between the penetration velocity computed and the one measured with x-rays. Both match remarkably well.

A sensitivity study of the different constitutive parameters was also performed to learn which parameters influence substantially on the penetration velocity. It was found that the results are only slightly sensitive. In fact the most sensitive parameter was found to be Y_0, the strength at zero pressure, if the impact velocity is slow enough

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