23^{RD} INTERNATIONAL SYMPOSIUM ON BALLISTICS
TARRAGONA, SPAIN 16-20 APRIL 2007

TETRAHEDRAL ELEMENTS FOR EXPLICIT BALLISTICS SIMULATIONS

A.J. Burton[1] and R.A. Clegg[1]

[1]Century Dynamics Ltd, Dynamics House, Hurst Road, Horsham, West Sussex, RH12 2DT, UK, Phone: +44 (0)1403 270066, Fax: ++44 (0) 1403 270099, email: Andrew.Burton@ansys.com, Richard.Clegg@ansys.com

In recent years there has been much interest in the use of tetrahedral elements in explicit numerical simulations. The primary motivation for this is the widespread availability of technology capable of automatically meshing complex geometries using tetrahedral elements. In comparison, meshing of complex geometries with 8-noded hexahedral elements is a skilled and time-consuming task that has not yet been fully automated. It is well known that constant strain linear tetrahedral elements exhibit locking behaviour under both bending and constant volumetric straining, for example, during plastic flow. This means that for explicit dynamic analyses these elements are best used in small numbers as "filler" elements in meshes dominated by hexahedral elements. An improved 4-noded element formulation, referred to as the average nodal pressure (ANP) tetrahedron, extending the work published by Burton [1] and Bonet [2,3], has been implemented in ANSYS® AUTODYN®. This robust and efficient element, with unique capabilities, can be used as a majority element in a mesh because it overcomes problems of volumetric locking for explicit dynamic problems, including those involving extreme strain, fragmentation and failure. In this paper, the performance of the ANP tetrahedral element formulation described above is illustrated for a range of terminal ballistics applications.

INTRODUCTION

When modelling complex 3D solid geometries for use in finite element analyses the generation of high quality hexahedral meshes can be a very time consuming process that only experienced CAE software users can achieve to a satisfactory standard. Over recent years, the increase in quality, efficiency and robustness of automatic tetrahedral mesh generators to represent complex 3D solid geometries has lead to the use of tetrahedral meshes as an attractive alternative to the more commonly used single Gauss point hexahedra.

It is widely known, however, that the linear 4-noded standard constant pressure (SCP) tetrahedron element suffers from volumetric locking for incompressible situations such as plastic flow, which leads to an “over-stiff” response in a finite element solution. This is easily explained by considering that there are generally more than 5 times the number of tetrahedral elements than hexahedral elements in a mesh with the same number of degrees of freedom. In an incompressible situation this leads to many more volume constraints in the tetrahedral problem and hence, the tendency for volumetric locking to occur.

To overcome these problems of volumetric locking an averaged nodal pressure (ANP) tetrahedron, first developed by Burton [1] and Bonet [2,3], has been implemented in the ANSYS AUTODYN software. This element calculates volumetric quantities at the nodes of the mesh leading to averaged nodal pressures, which are subsequently averaged back to the elements and integrated to give equivalent internal nodal forces.

The element has been further enhanced from the original work undertaken by Burton [1] and Bonet [2,3] in which it was employed in a hyperelastic framework only. The functionality of the ANP tetrahedron has been extended to allow it to be used with linear, hyperelastic, Mie-Gruniesen energy dependent and porous equations of state allowing a wide range of materials, such as metals, rubbers, ceramics and concrete to be represented in simulations involving shock propagation.

With the ANP tetrahedral element deviatoric strains and consequently deviatoric stresses are calculated in the same way as a standard linear tetrahedron. In order to calculate the volumetric strains, for a linear equation of state, nodal volumes are first calculated by assembling components from the elements surrounding a node as [1,2,3];

$$v_a = \sum_{e=1}^{m_a} \frac{1}{4} v_e \tag{1}$$

The averaged nodal strain rates can be calculated by using a volume weighted approach as follows [4];

$$\dot{\varepsilon}_a = \frac{1}{v_a} \sum_{e=1}^{m_a} \frac{1}{4} v_e \dot{\varepsilon}_e \tag{2}$$

Nodal variables such as volumetric strain, density and compression are updated using an incremental approach leading to the calculation of an averaged nodal pressure. Once the nodal pressures have been calculated these are averaged back to the elements to give the averaged element pressure as [1,2,3];

$$\overline{p}_e = \sum_{a=1}^{4} \frac{1}{4} p_a \tag{3}$$

The average element pressure is added to the standard deviatoric stresses to give the total stress state in an element at the current time step.

The ANP tetrahedral elements implemented in the ANSYS AUTODYN software can be used in conjunction with a large number of the available equations of state, failure models, automatic erosion, interaction and Euler-Lagrange coupling algorithms. This enables a large range of complex 3D applications to be simulated with the element.

NUMERICAL EXAMPLES

A series of numerical examples are presented hereafter to assess the accuracy and robustness of the ANP tetrahedral formulation. Results have been compared to experimental results and also to analyses using alternative element formulations, such as hexahedra, standard linear constant pressure (SCP) tetrahedra and Smooth Particle Hydrodynamics (SPH) solvers.

Taylor Test

A particular validation test carried out using the ANP tetrahedral element was a Taylor test involving the impact of an iron cylinder onto a rigid wall. The cylinder had an original length of 30mm and diameter 10mm with an impact velocity of 221m/s. Material data was taken from the standard AUTODYN material library and the rigid wall was simulated using velocity boundary conditions applied to one end of the cylinder. Plots of effective plastic strain at a time 0.05ms for three element formulations are shown in Figure 1. The left hand plot shows the result for the SCP tetrahedra which is susceptible to volumetric locking. When compared to the hexahedral elements shown on the right hand side it can be seen that the effective plastic strains at the centre of the cylinder are lower for the SCP tetrahedral element. However, plastic strains for the ANP tetrahedral formulation, shown in the centre of the plot, compare extremely well to the results for the hexahedral elements. This is further confirmed by the summary of final length and diameter given in

Table 1, where results are also compared to experiment [5]. Results for the ANP tetrahedra compare well to both hexahedral elements and experiment, whereas the final diameter of the SCP tetrahedra is much lower confirming that numerical locking has taken place for this element formulation.

Table 1 – Summary of Experimental and Numerical Taylor Test Results

	Experiment [5]	ANP tet.	Hex.	SCP tet.
Cylinder length (mm)	**23.13 to 23.59**	**23.28**	**23.3**	**23.31**
Impact Diameter (mm)	**16.70 to 17.04**	**16.89**	**16.76**	**16.01**

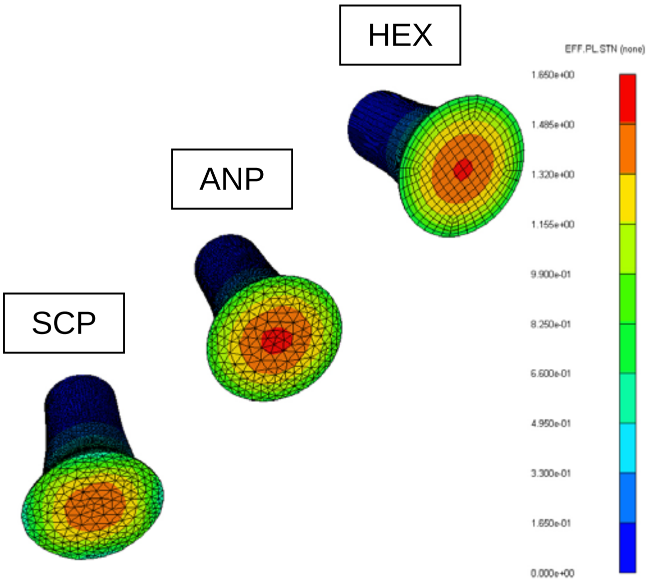


Figure 1 – Effective Plastic Strain at time 0.05ms

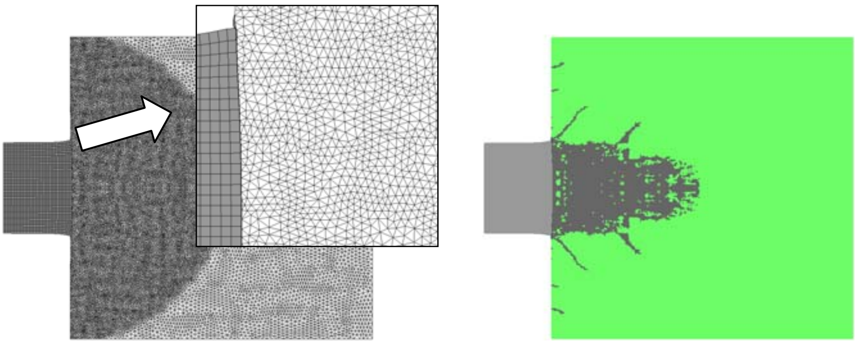
Ceramic Edge on Impact

Simulation of an edge on impact experiment onto a SiC target reported by Strassburger [6] and previously studied by Century Dynamics [7] and EMI [8] has been undertaken. The experiment consisted of a cylindrical steel projectile (r=15mm, L=23mm) impacting the edge of a SiC target (100mmx100mmx10mm) at 185m/s.

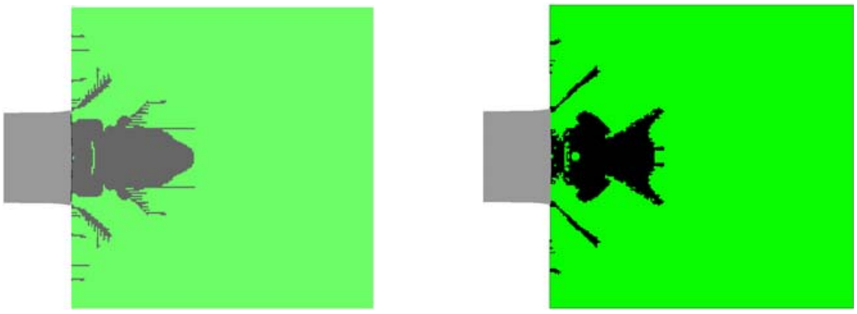
Three numerical analyses were performed, assuming quarter symmetry about the axis of impact. In each analysis 3D hexahedral elements were used to represent the steel projectile whilst the SiC target was represented in turn by SPH (200000 particles), hexahedra (200000 elements, 250416 nodes) and ANP tetrahedra (540378 elements, 100182 nodes). For the tetrahedral mesh automatic mesh generators were utilised enabling a finely zoned region to easily be created around the region of impact, as shown in Figure 2a.

Interaction between the projectile and target was achieved using the AUTODYN gap contact algorithm. Material data for the SiC was taken from the standard AUTODYN material library, derived from data given by Holmquist [9].

Results at 4.6μs after impact are shown in Figure 2b,c,d for the 3 analyses. Comparison between crack patterns and the experiment [6] show good correlation for all 3 solver types. In addition, the ANP tetrahedral elements perform well compared to both the hexahedral and SPH solutions.



(a) ANP mesh **(b) ANP tet. crack patterns**



(c) Hex. crack patterns **(d) SPH crack patterns**

Figure 2 – Ceramic Edge on Impact Simulation results at time 4.6μs

Steel Ball Impact onto Ceramic

Numerical simulations of the impact of a 6.35mm diameter steel sphere onto a 25mm thick Sintox-FA (95% pure) alumina target at 1449m/s have been performed. Results have been compared to experiment [10] and also previous numerical studies with AUTODYN using the 2D SPH solver [7,11]. For the 3D analyses a quarter symmetry model was utilised and the tile was assumed to be circular to allow direct comparison with 2D axisymmetric results [7,11]. The Sintox-FA material was represented using the JH2 model with crack softening, where input data was derived from [10] and [12]. An instantaneous geometric strain erosion model was also used for the Sintox-FA with an erosion strain set at 1.0. For the steel ball and outer support frame of the target a Johnson Cook strength model and shock equation of state were used. Two analyses were performed with hexahedra (907200 elements, 935557 nodes) and ANP tetrahedra (1574743 elements, 270072 nodes) used for the target respectively. The tetrahedral mesh is shown in Figure 3, where automatic mesh generators have been used to create a finely zoned region around the impact zone. In both cases SPH particles were used to represent the steel ball.

Results for the hexahedral and ANP tetrahedral targets are shown in Figure 4. The crack patterns and overall distribution of damage compare well to each other and also with previous studies using the 2D SPH solver [7,11]. Crack growth on the hexahedral model, however, does show a tendency to follow the grid lines through the mesh. In addition when compared to the experimental results, [7,11], the simulated depth of penetration for the ANP tetrahedral mesh (6.6mm) compares extremely well to experiment (6.1mm) whereas the hexahedral model gives a larger value (8mm). Note, however, that the erosion strain of 1.0 used in this analysis is at the lower end of the range generally used with hexahedral elements in 3D AUTODYN analyses. The hexahedral results could therefore be improved by using a higher value for the erosion strain.

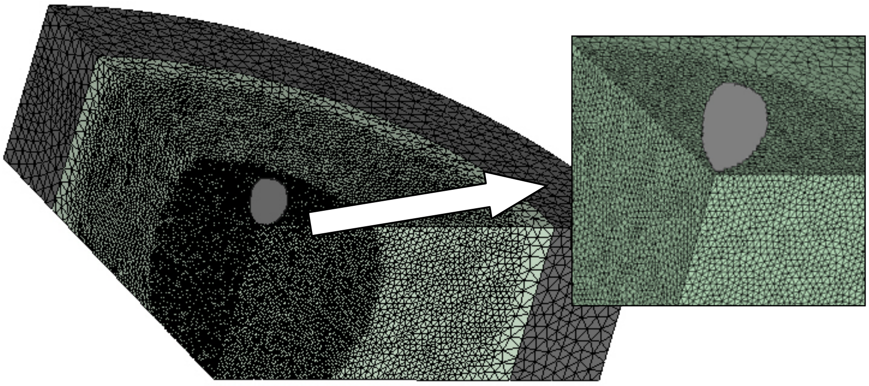
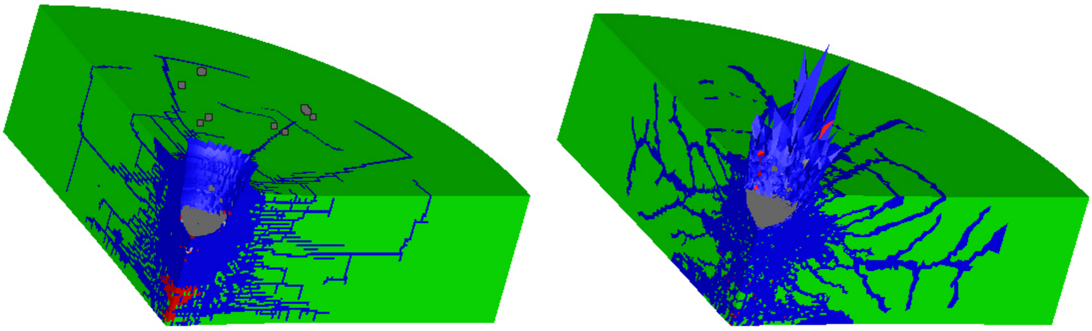


Figure 3. ANP tetrahedral mesh containing 1574743 elements



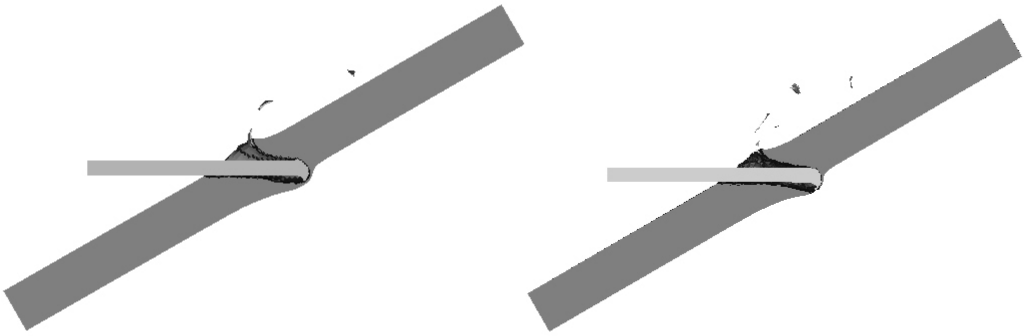
(a) Hexahedra **(b) ANP tetrahedra**

Figure 4. Crack Patterns at time 0.02ms

Impact of a Tungsten Rod onto RHA

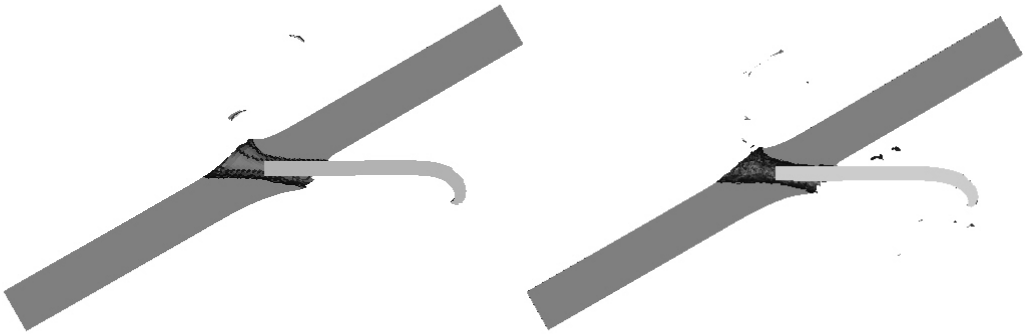
This analysis is based on experimental and numerical studies undertaken by EMI [13]. A cylindrical tungsten rod of length 130mm and radius 6.5mm is impacted at 1600m/s onto the centre of a rectangular RHA target (250mmx120mmx20mm). Material data for Tungsten and RHA were built from data contained in the standard AUTODYN material library. Two analyses were undertaken in order that results from hexahedral (111020 elements, 136752 nodes) and ANP tetrahedral (823255 elements, 145501 nodes) models could be compared directly. Results for both models are shown in Figure 5 and Figure 6 at two stages during the analysis. Penetration depth, projectile shape and target damage all compare well between the two solver types.

Note, the results shown here are different to those presented in [13] due to differences in the failure models used. Here we focus on comparing two numerical formulations rather than against experiment.



(a) Hexahedra **(b) ANP tetrahedra**

Figure 5. Cylindrical Tungsten Rod Impact onto RHA, time 0.05ms



(a) Hexahedra **(b) ANP tetrahedra**

Figure 6. Cylindrical Tungsten Rod Impact onto RHA, time 0.1ms

Ballistic Impact onto Multi-Layered Armour

This final example demonstrates the capability of the ANP tetrahedral element to be used in a more complex analysis of a complete armour system. A multi-layered target consisting of a ceramic and Kevlar-epoxy core, sandwiched by two layers of aluminium is impacted obliquely by an armour piercing projectile at 894m/s. All material data is taken from the standard AUTODYN material library.

The layers of the target are separated by steel washers and held together with steel bolts. ANP tetrahedral elements are used to represent the projectile and bolts whilst the armour and washers are modelled using hexahedral elements (total elements 306657, total nodes 237215). A finely zoned region is created around the region of impact. Contours of damage on the ceramic layers during the analysis are shown in Figure 7.

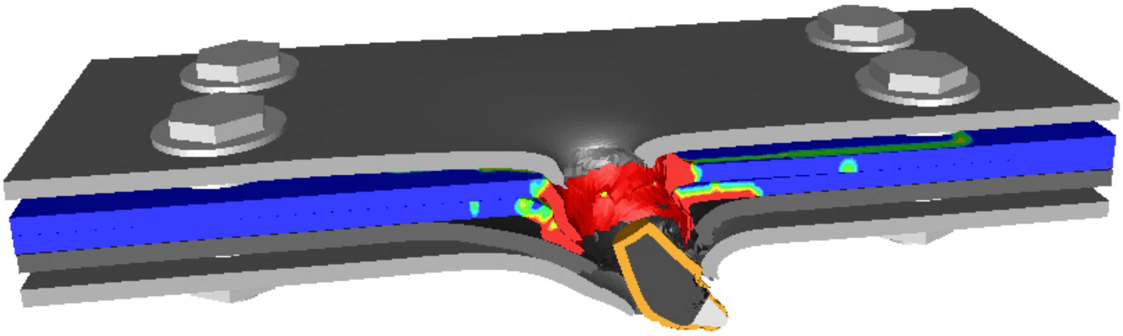


Figure 7. Ballistic Impact onto Multi Layered Armour

CONCLUSION

An improved linear tetrahedral element formulation for explicit dynamic applications, referred to as the average nodal pressure (ANP) tetrahedron, has been implemented in ANSYS AUTODYN. The element extends work undertaken by Burton [1] and Bonet [2,3] and is robust and efficient, with unique capabilities. It can be used as a majority element in a mesh because it overcomes problems of volumetric locking that occur with a standard linear tetrahedron element. The element can be applied to explicit dynamic applications, including those involving extreme strain, fragmentation and failure.

In this paper validation tests on a range of materials under ballistic impact conditions have been presented, highlighting the potential for the use of tetrahedra in ballistic impact simulations on complex armour systems when geometry constraints or time required creating a hexahedral mesh becomes prohibitive. The ANP tetrahedra can be used in conjunction with other element formulations, such as hexahedra and SPH, and also coupled with automatic erosion, failure and interaction algorithms.

REFERENCES

[1] A.J. Burton, Explicit, Large Strain, Dynamic Finite Element Analysis with Applications to Human Body Impact Problems, *PhD Thesis*, University of Wales, (1996).

[2] J. Bonet, A.J. Burton, A simple average nodal pressure tetrahedral element for incompressible and nearly incompressible dynamic explicit applications, *Communications in Numerical Methods in Engineering*, **14**, 437-449, (1998).

[3] J. Bonet, A.J. Burton, A simple average nodal pressure tetrahedron for nearly incompressible dynamic applications", *Euromech 371, Efficient and Reliable Finite Elements for Linear and Non-Linear Analysis*, Karlsruhe, Germany, (1997).

[4] J. Bonet, H. Marriott, O. Hassan, An averaged nodal deformation gradient linear tetrahedral element for large strain explicit dynamic applications, *Communications in Numerical Methods in Engineering*, **17**, 551-561, (2001).

[5] C.J. Hayhurst, R.A. Clegg, I.H. Livingstone, N.J. Francis, The Application of SPH techniques in AUTODYN-2D^{TM} to Ballistic Impact Problems, *16th International Symposium on Ballistics*, San Francisco, CA, (1996).

[6] E. Strassburger, H. Senf, Experimental Investigation of Wave and Fracture Phenomena in Impacted Ceramics, EMI report 3/94.

[7] R.A. Clegg, C.J. Hayhurst, I. Robertson, Development and Application of a Rankine Plasticity Model for Improved Prediction of Tensile Cracking in Ceramic and Concrete materials under Impact, *14th DYMAT Technical Meeting*, Sevilla, Spain, (2002).

[8] S. Hiermaier, W. Riedel, Numerical Simulation of Failure in Brittle Materials using Smooth Particle Hydrodynamics, *International Workshop On New Models and Numerical Codes for Shock Wave Processes in Condensed Media*, St Catherine's College, Oxford, UK (1997).

[9] T.J. Holmquist, G.R. Johnson, response of Silicon carbide to High Velocity Impact, *In Proc of Structures under Shock and Impact*, (2000).

[10] P.J. Hazell, The Failure of Ceramic Armour Subjected to High Velocity Impact, *EngD Thesis*, Royal Military College of Science, UK, (1998).

[11] R.A. Clegg, C.J. Hayhurst, Numerical Modelling of the Compressive and Tensile Response of Brittle Materials Under High Pressure Dynamic Loading, *Shock Compression of Condensed Matter*, Snowbird, USA (1999).

[12] C.E. Anderson, G.R. Johnson, T.J. Holmquist, Ballistic Experiments and ……, *15th International Symposium on Ballistics*, Israel, (1995).

[13] N. Heider, K. Weber, P. Weidermaier, Experimental and Numerical Simulation Analysis of the Impact Process of Structured KE Penetrators onto Semi-Infinite and Oblique Plate Targets, *21st International Symposium on Ballistics*, Adelaide, Australia, (2004).