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THE MAXIMUM MACH NUMBER OF COHERENT COPPER JET

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It is well known that the velocity of a coherent jet emerging from a shaped charge is limited. For copper liners, the maximum Mach number for the flow into a coherent jet is 1.23. This value was first found experimentally on 1974. Many attempts were made to explain it. We hereby present a simplified model that derives this value from first principles. Surprisingly it is found that the main reason for this limit is geometrical. The conditions for the emergence of coherent copper jet at this upper limit are analyzed. Predictions for other metals are presented as well.

INTRODUCTION

On 1974 Harrison, DiPersio, Krapp and Jameson showed that when the flow velocity of the collapsing copper liner becomes larger than $1.23C_0$, where C_0 is the copper bulk sound velocity, the formed jet becomes incoherent. In this case, instead of a straight coherent jet one may get a spray of laterally expanding jet particles (called sometimes bifurcation due to its X-ray shadow) [1]. This observation has since then been verified by many investigators [2-10]. Following the attempts to explain this result Walker summarized [11]: "The use of hydro-codes allows the artificial alternation of material properties to examine directly the role of material properties in incoherence, but still more work is needed in explaining the origin of the 1.23 multiplier factor." Some researchers postulated that the 1.23 factor is not a constant, and might be a specific case of a more general theory. In works such as [12] the idea was to find a fit to the available data by using a formula suggested by the authors that was based on some physical assumptions. The formula they reached was based on the material compressibility but it did not lead to an explanation for the 1.23 factor.

THE ONE STREAMLINE MODEL

The One Streamline (OS) Model is the basis for our new and generalized model that predicts the 1.23 flow factor. The OS model, introduced by Hirsch on 1983 [13] and verified by simulations on 1987 [14] for incompressible strength-less fluids is applied below. This model was extended on 1995 by N. Heider [15] to include the material strength effect, but the test he suggested for his hypothesis did not lead to conclusive results. The OS model was also extended to the planar non-symmetric flow problems (see e.g. [16]). Hirsch and Yossifon [17] applied the model to explain the failure mechanism observed in the penetration of metallic targets near the ballistic limit, where for the first time a strong supportive evidence for the role of the strength and the compressibility in the flow was presented.

The simplest example described by the model is the turn of the flow at a 90˚ corner of infinitely hard walls as shown in Fig.1a for the real flow and in Fig.1b for the OS model. This flow description by the OS model is a simplification of the real Bernoulli flow shown in Fig.1a. The curvature radius of the streamline in the model, which is constant along the whole turn, is equal to the curvature radius of the real flow at the bisector (here at 45˚). The stagnation pressure, existing in the real flow at one point, is assigned in the model to the whole volume that is confined by the streamline. As a result, the confined volume in the model is smaller than that in the real flow. The stagnation pressure P_S , for an incompressible strength-less material, is equal to $.5\rho_0 V^2$. This pressure is balanced by the centripetal force of the turning flow: $P_S = H\rho_0 V^2 / R$, where *H* is the incoming flow width and *R* is the streamline curvature radius, measured from the turn center to the virtual interface line between the stagnant volume and the one streamline. By equating the two expressions for the stagnation pressure we get the geometrical relation: *R = 2H*. This result was confirmed employing hydro-code-simulations [14].

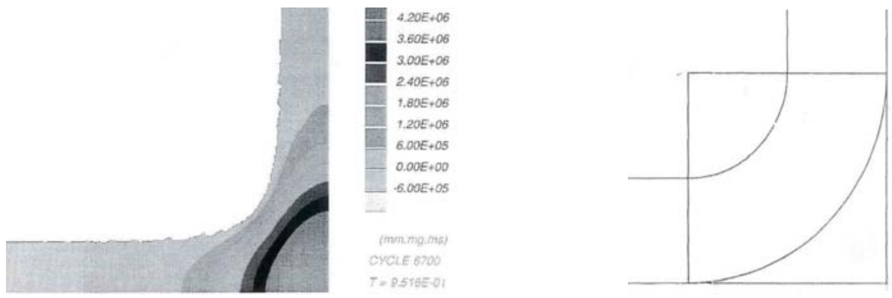

Figure 1. (a) The flow field of incompressible fluid around a 90° corner (left). (b) The OS model for the same flow (right).

EFFECT OF SUPERSONIC COLLISION ON THE FLOW

The flow of copper into an infinitely hard wall at 2000m/sec (Fig.1a) is now compared to the flow at 9500m/sec shown in Fig.2. We observe that the subsonic flow is symmetric about the bisector. In the supersonic flow there is a fast rise of the pressure when the flow comes in into the plastic flow region (the vertical broken line). Then the pressure rises as in a Bernoulli like flow. On the outgoing flow side the high pressure extends further than in the subsonic flow. The free surface curvature radius at the supersonic flow is much smaller at the bisector than this curvature radius at the subsonic flow. The flow thickness monotonically reduces at the subsonic flow as it leaves the high-pressure region. At the supersonic flow however, the flow thickness reaches a minimum followed by an expansion that breaks the flow into many particles. The copper having an initial strength of 2.9Kbar loses its strength at a strain of 0.5 (which is equivalent to 145 Joul/cm^3) during the incoming flow [18]. The location where the material strength is lost, called the strength failure surface (SFS, [18,19]) is where the flow direction begins to change, which is about one incoming flow thickness away from the stopping hard wall.

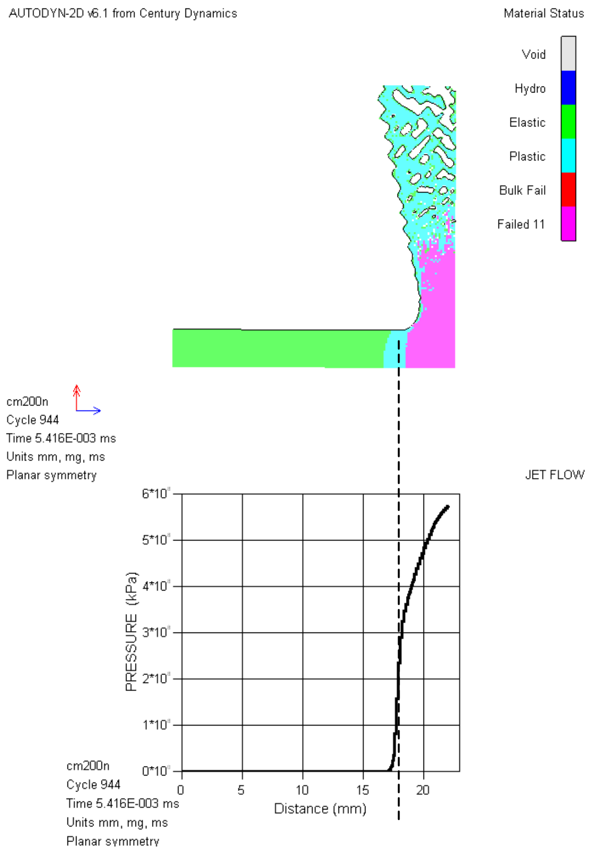

Figure 2. Flow of copper along a 90 degrees corner at 9500m/sec.

As the flow velocity increases and gets close to the bulk sound velocity C_0 several phenomena occur. The material compressibility causes the stagnation pressure P_S to increase above the Bernoulli pressure $.5\rho_0 V^2$ for an incompressible material, becoming equal to $.5\rho_S V^2$, since the mass density ρ_S, of the compressed material at the stagnation point is larger than ρ_0.

Following the shock, the flow thickness rises from H to H' where $H' = HV / C_0$ and the flow velocity reduces to C_0. The stagnation pressure thus rises now to the value:

$$P_S = \rho_0 (V - C_0)V + .5\rho_S C_0^2 \tag{1}$$

To maintain the flow balance the centripetal force, now equal in the OS model to $H' \rho_0 C_0^2 / R$ must be also equal to P_S. We thus get

$$P_S = \rho_0 (V - C_0)V + .5\rho_S C_0^2 = H' \rho_0 C_0^2 / R \tag{2}$$

Note that in the one streamline model the mass density remains ρ_0 in the streamline and the standing shock is treated as if being sharp. The reason why the standing shock is not sharp in the real flow close to Mach one is the fact that the swell of the flow from H to H\` takes time to occur due to the flowing material inertia.

THE INCOHERENCE CONDITION IN SUPERSONIC FLOW

To calculate the material mass density at the stagnation point we have to apply its equation of state. Here we use the Murnaghan E.O.S. [20] for materials compression formulated as in [12]:

$$P(\rho) = (K_0 / n) \cdot [(\rho / \rho_0)^n - 1] \tag{3}$$

where K_o is the bulk modulus and n is a constant of the material. Balancing the stagnation pressure and centripetal force we have:

$$P_S = (H' / R)\rho_0 C_0^2 = K_0 / n \cdot [(\rho_S / \rho_0)^n - 1] \tag{4}$$

Changing terms we get for the stagnation point mass density:

$$\rho_S = \rho_0[1 + P_S n / K_0]^{1/n} \tag{5}$$

Substituting into Eq. 2 we get:

$$P_S = (H^{'} / R)\rho_0 C_0^2 = .5\rho_0[1 + P_S n / K_0]^{1/n} C_0^2 + \rho_0(V - C_0)V \tag{6}$$

Solving this quadratic equation in V / C_0 for this Mach number we get:

$$V / C_0 = .5 + \{.25 + H^{'} / R - .5[1 + (H^{'} / R)N\rho_0 C^2 / K_0]^{1/n}\}^{1/2} \tag{7}$$

It is clear that if R becomes smaller than $H^`$ to keep the balance between $P_S$ and the centripetal force the flow cannot remain coherent (see also [17]). Applying this geometric coherency limit condition of *H`/R* = 1 and identifying the bulk modulus as $K_0 = \rho_0 C_0^2$ we finally get:

$$V_{COHERENCY\ \ LIMIT} / C_0 = .5 + \{1.25 - .5 \cdot [1 + n]^{1/n}\}^{1/2} \tag{8}$$

For Copper, Molybdenum, Aluminum, and Nickel the values of *n* are respectively 4.99, 4.01, 4.54, and 4.74 [12]. The limiting Mach numbers that Eq.8 predicts for these materials respectively are: 1.231, 1.209, 1.222 and 1.226.

From Eq.7 we get the Mach number as a function of H'/R as shown in Fig.3. The stagnation pressure is given by Eq.2. For linearly compressible material, *n* = 1, we would get that the maximum coherent Mach number is 1. It means that a coherent jet can be formed in supersonic flow because the compression of real materials usually follows the Murnaghan law (Eq.3 and [12,20]) with the power constant *n* > 1.

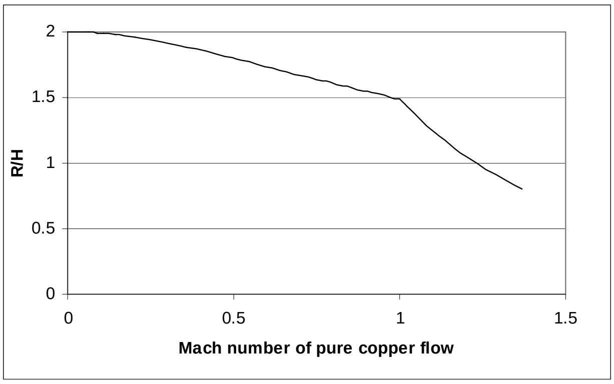

Figure 3. The predicted radius R/H' as a function of the Mach number.
(The R/H' region close to one cannot be reached by the real flow).

COMPARISON WITH SIMULATIONS

The above analysis applies both to planar and axially symmetric flow. It is clear that in the planar geometry complete symmetry exists about the bisector (at 45º) between the incoming and outgoing flow sides, when $V << C_0$ and the material strength is negligible.

This symmetry about the bisector is lost as soon as the flow speed increases above Mach 1, due to the formation of the standing shock region. Close to $H'/R = 1$ the flow is physically unable to form the small curvature radius turn needed to keep the balance between the stagnation pressure and the centripetal force. That causes the shape of the outgoing flow side to change. The pressure release at the outgoing flow takes longer and the mass density of the flowing material is still high when being already out of the region affected by the centripetal force. When the elastic energy accumulated in the compressed material is released without the balancing effect of the centripetal force, then this energy release gives rise to a high expansion velocity component in the outgoing flow. This lateral expansion velocity causes the jet to disintegrate. The only force acting to prevent the flow disintegration by its expansion is the material adhesion.

In Fig.4 we compare the flow of copper at Mach 1.23 for the three cases where the adhesion forces are 1, 20 and 100 Kbar. The effect of the adhesion force is immediately observed. The same effect is seen in Fig.5 where the flow runs have the same respective parameters in the axial symmetry.

In the real conditions of jet tip formation there is usually not enough time for the flow to reach steady state conditions as in the examples shown here. Therefore, in order to perform a real measurement of the adhesion force prevailing in copper during the transient period of the jet tip formation it is necessary to do an exact simulation of the specific experiment used. In such a simulation the addition of the explosive products pressure to that of the centripetal force of the flow is taken into account. This added component is not large but it may still be of some influence on the measured results. Moreover, when we observe in X-rays a perfectly coherent jet it may already have some degree of internal formation of cavities, occurring in axial symmetry especially near the axis. This cavity formation may absorb part of the expansion energy.

SUMMARY AND CONCLUSIONS

The loss of the outgoing flow coherency mainly depends on two factors. The material equation of state, that determines at what Mach number the condition $H'/R = 1$ is reached and the force of adhesion that prevents the material from disintegrating for as long as it can contain the elastic energy released by the material at its expansion phase. Since the adhesion in the low temperature melting metals is usually low, they generally

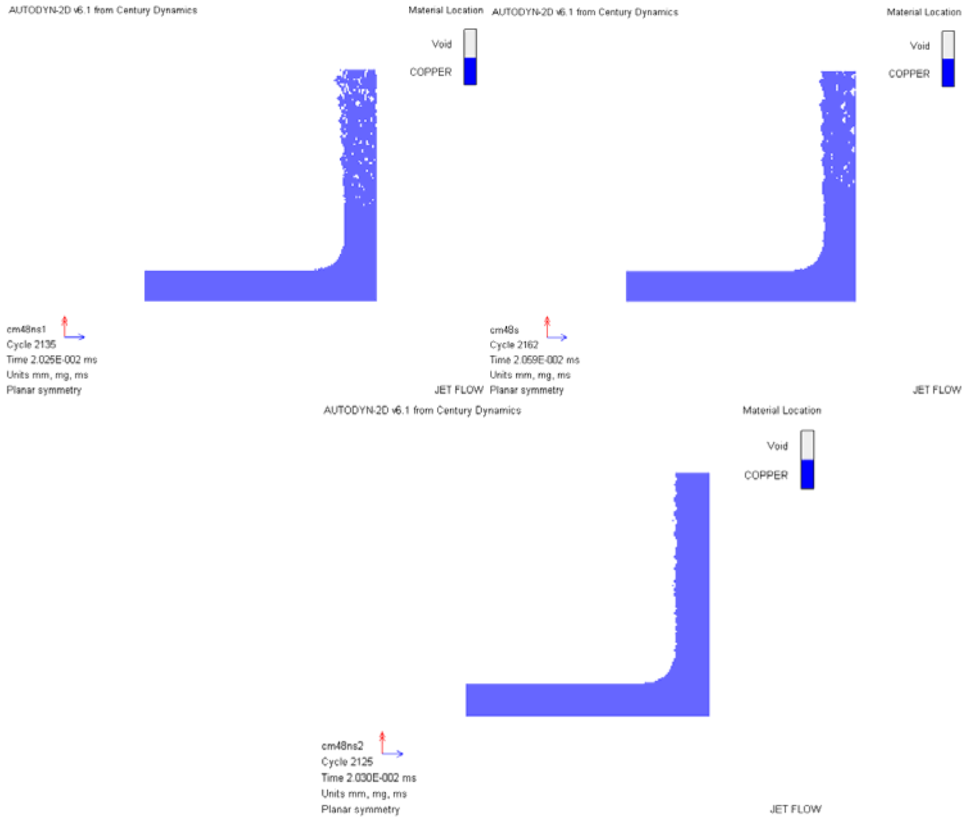

Figure 4. Flow at planar symmetry of copper at Mach 1.23 with adhesion of 1Kbar (top left), 20 Kbar (top right) and 100 Kbar (bottom).

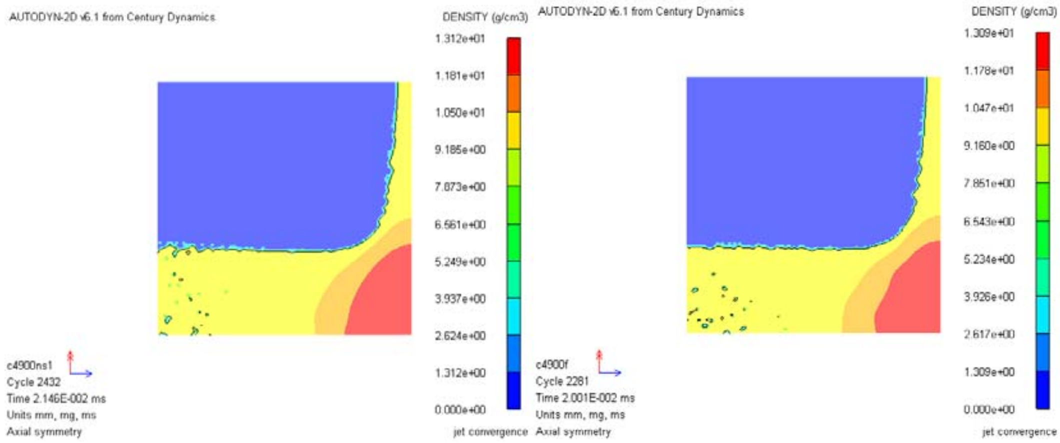

Figure 5. Cylindrical symmetry flow (jet entering at the top) of copper at Mach 1.23 with adhesion of 20 Kbar (left) and 100 Kbar (right). Note the concentration of the cavitations close to the axis of symmetry.

form non-coherent jets considerably below the H'/R = 1 limit. When the experimentally measured coherency limit is known, it is possible to use the simulation for assessing the adhesion force prevailing during the short time when the shaped charge jet tip is formed. The internal cavities formation near the jet tip center should also be considered.

In a real shaped charge jet the explosive products pressure adds to that of the centripetal force. This may cause a small increase in the Mach number coherency limit comparing to the case where there is no such gas pressure support. Therefore a correct simulation of the formation of a shaped charge tip close to the coherency limit must take this additional pressure into account. Influence of the liner material viscosity may also be a factor influencing the results, though it was not included in the current simulations.

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