EXPLOSIVE ACCELERATION OF FRAGMENTS AS A FUNCTION OF THEIR POSITION RELATIVE TO THE EXPLOSIVE SURFACE

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A two stage model for the acceleration of fragments by explosive warheads is proposed to approximate the velocity reached by spherical metal fragments placed inside or outside a symmetrically detonated spherical charge. Experiments conducted by M. Held for placement of a fragment near the base of a cylindrical charge suggest that the maximum velocity is obtained for partially embedded fragments. We replicated this result by Autodyn simulations, and obtained a similar result with the two-stage model. The first stage is acceleration due to wave diffraction at the spherical fragment surface. The second stage is continuous acceleration by drag force due to entrainment of the fragment by the expanding detonation products. We approximate the products flow field as having linear velocity distribution, and a more elaborate mass-conserving density distribution.

INTRODUCTION

Controlled fragment acceleration is of major interest in warhead ballistics. A simple estimate of the fragment final velocity is readily obtained by one of the Gurney formulae [1]. This approximation is valid only for fragments resulting from a confining liner where the fragmentation takes place at the end of the acceleration process. The Gurney model is based on a simplified expansion flow field, assuming linear velocity and uniform density distributions. Invoking energy and momentum balance leads to an estimate of the asymptotic liner velocity that in the spherical case reads

$$V_g = \frac{V_E}{(M/C + 0.6)^{1/2}} \tag{1}$$

Where V_E is a characteristic velocity related to the specific energy of the explosive, and M, C are the mass of the liner casing and the charge respectively. Obviously, a Gurney model cannot predict the terminal velocity of a single non-confining fragment. There are other approaches to estimating the velocity of a confining liner as a time-space evolution process [2-3], but they are of less practical use.

It is well known that the acceleration of a single fragment strongly depends on its initial location relative to the charge surface, an effect that cannot be accounted for by the Gurney model. The first experimental data that shows this dependency is due to Manfred Held [4] and is presented in Figure 1.

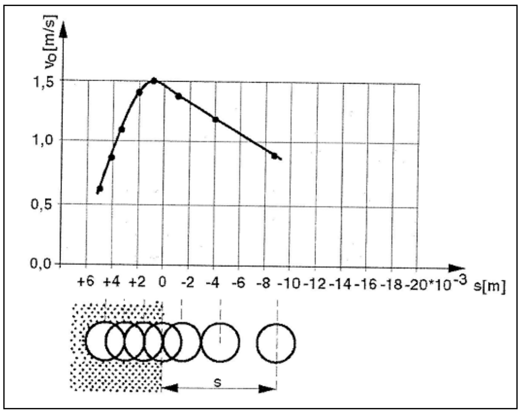

Figure 1. Experimental data for the final velocity of a 3mm diameter steel sphere from the base of a cylindrical charge vs. its initial position, published by M. Held [4].

This data clearly shows that the terminal velocity of a 3mm steel sphere reaches a peak value when it is partly embedded in the explosive. When the sphere is placed deeper in the explosive, or at a standoff location outside, its final velocity decreases.

There are few publications in the open literature concerning the acceleration of a single fragment. Some of them are experimental (mainly by M. Held), and some are theoretical, see for example references [5-6]. Nebenzahl [5] estimated the velocity vector of a surface-embedded sphere fragment due to a sliding detonation by a simple model for the diffraction of the detonation front at a partially-embedded sphere. Duvall and his colleagues [6] calculated the velocity of a small projectile accelerated by a plane layer of explosive assuming acceleration by a constant-coefficient drag force. Their results indicate that a maximum projectile velocity is obtained for an outer placement, in disagreement with Held's experimental results.

In our study we model the fragment acceleration by a detonated charge, seeking an agreement with the placement-velocity trend observed by Held. The configuration we consider is a symmetrically detonated spherical charge with a small spherical fragment placed inside or outside the charge. To assist in understanding the acceleration mechanism we used the Autodyn numerical code [7]. We first outline the main results of these simulations, and then proceed to describe our model for the acceleration process. As a preparatory stage, we computed the terminal velocity of the sphere in Held's experiment, for s=-5mm, 0 and 9mm. The results were close to those measured by Held, hence this test constitutes a rough validation of the Autodyn scheme for the present study.

GENERAL CONSIDERATIONS

Let a small metallic sphere be embedded in a spherical explosive charge initiated at its center. Using the Autodyn CEL (Coupled Euler Lagrange) option, the sphere trajectory is obtained solely from the constitutive laws of the solids and gas involved. Inspecting these simulations, we tracked pressure, density and velocity profiles in the products following the detonation of the spherical charge as shown in Fig. 2 (without the interference of the fragment).

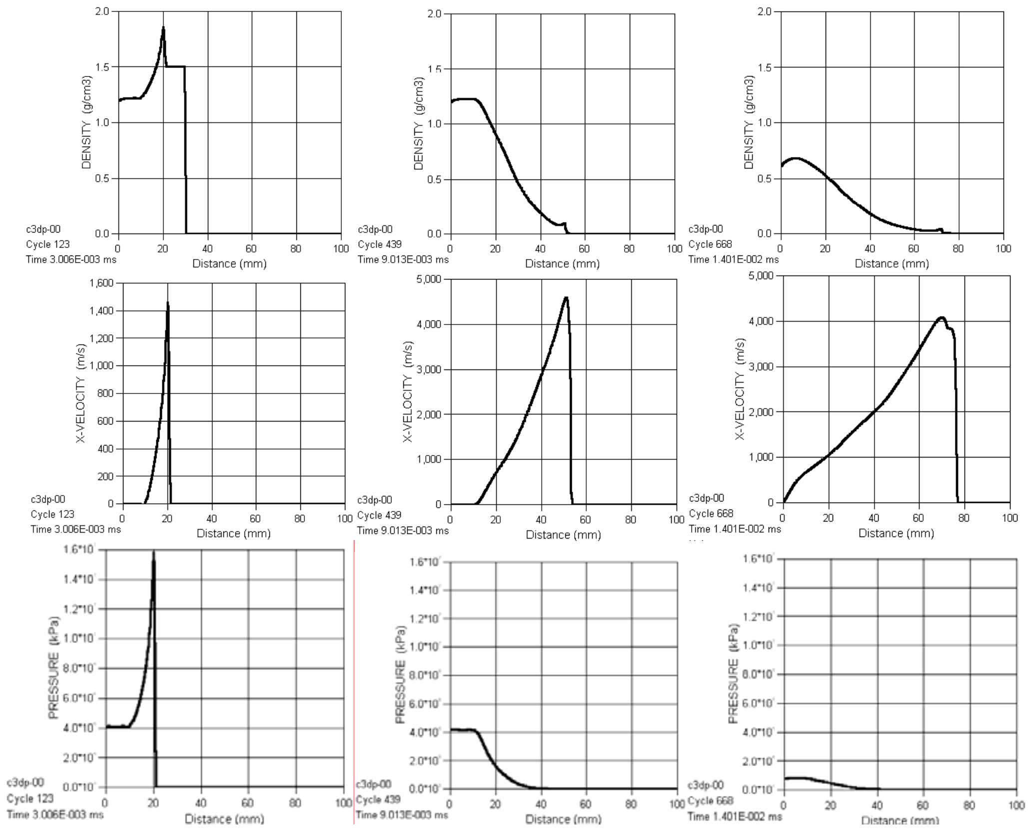

Figure 2. Density, velocity and pressure profiles in the detonation products for a sphere detonated in ambient pressure air, at 3, 9 and 14µs.

This data demonstrates that the underlining assumptions of the Gurney formulation do not hold in this case, so that the Gurney estimate of the terminal velocity of the metallic sphere is invalid here.

Referring to Fig. 2 for a TNT charge of $R_C = 30mm$, the spatial distributions are shown at times $T_1 = 3\mu s$ (before completion of the detonation), $T_2 = 9\mu s$ and $T_3 = 14\mu s$ (after completion of the detonation). At T_1 the fluid velocity and density behind the detonation front are $u_{CJ} \approx D/4$ and $\rho_{CJ} \approx \frac{4}{3}\rho_0$, where D is the detonation velocity and ρ_0 is the initial density of the explosive (assuming $\gamma = 3$). As the charge is completely detonated, the pressure and the density drop while the velocity of the expanding detonation products increases sharply. In free-expansion mode (into surrounding air) the velocity peak decreases slowly in time while the density and pressure drop much faster. It is interesting to note that the velocity of the expanding gas is roughly linear with the radial coordinate, while the density displays an inverse trend.

We first consider the maximum-terminal-velocity case where the fragment is partially embedded, and then we proceed to examine the inner or outer placement cases.

SPHERE EMBEDDED WITHIN THE CHARGE

The simulations indicate the following "physical scenario" of the acceleration process:

(1) Upon the arrival of the detonation front it is reflected from, and diffracted around, the sphere, as can be seen in Fig. 3. The acceleration is positive until the

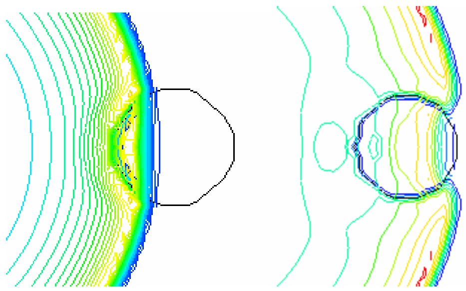

Figure 3. Isobars for the detonation wave diffracted by the sphere.

wave has somewhat traversed the spherical mid-plane. From then on the sphere is decelerated up until the end of the diffraction phase. This pattern is readily interpreted as follows. At the reflection-diffraction stage the upstream-facing part of the spherical surface is exposed to the high pressure accompanying the reflection of the detonation wave, in a similar way to the reflection of a planar shock wave. Unlike the case of a shock reflection, however, here the high pressure gradient behind the detonation front contributes a significant negative impulse once the front has passed the mid-plane, and the total force vanishes when about 2/3 of the sphere has been engulfed by the front. This explains why the peak velocity is obtained for a partly embedded sphere.

(2) The end of the wave-interaction phase is marked by the point of sign-reversal on the sphere velocity history, where the velocity has reached a local minimum value, as shown in Fig. 4. From that time on, the sphere is smoothly accelerated as it is entrained by the detonation products. Unlike steady entrainment by an oncoming flow, the acceleration in this case is not only by dynamic pressure (multiplied by a suitable drag coefficient), involving also an initial diffraction-phase velocity increment.

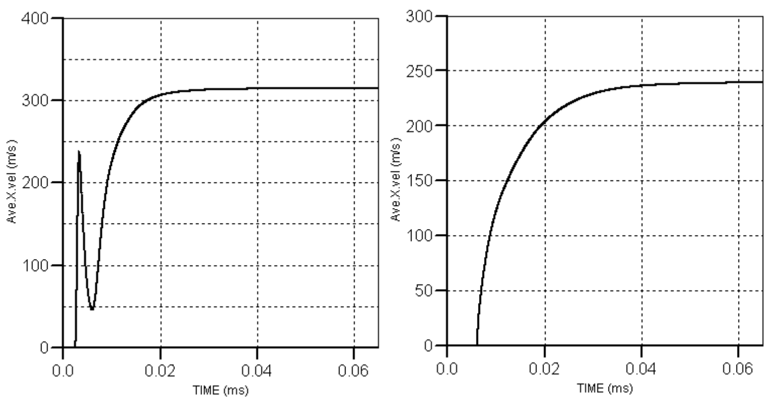

Figure 4. Velocity time-history of a sphere embedded in the explosive; left: S=10mm, and in air; right: S=-10, (see position in Fig.1).

We thus propose a two-stage acceleration model. The first stage is the lumped acceleration due to the entire process of wave diffraction around the sphere; in the second stage the sphere is smoothly accelerated by a drag force. We first present the diffraction phase, and follow by an outline of the drag phase.

MODEL DESCRIPTION

The diffraction model is formulated separately for the two different cases of internal and external placement. Turning to the internal case, the in-charge diffraction loading is modeled by the space-time surface pressure distribution [5],

$$P(t) = \begin{cases} P_{CJ} \cdot e^{-t/\tau_1}(1 + e^{-t/\tau_2}\cos\theta) & 0 \le \theta \le \pi/2 \\ P_{CJ} \cdot e^{-t/\tau_1} & \pi/2 \le \theta \le \pi \end{cases} \tag{2}$$

where θ=0 at the upstream-facing point. In the case of a zero-gradient detonation front, the surface pressure immediately behind the diffracting front would be approximated by Eq. 2 with $t = 0$ (or, alternatively, with infinite time-decay constants).

The time decay is characterized by two time constants τ_1 and τ_2, where the first refers to the decay of the pressure in the detonation products flow field, and the second refers to the equalization time of the reflected pressure due to the shock diffraction around the sphere. Since we seek the total time-integrated momentum on the sphere, the exponential decay factors do not have to include the deferred initial moment (of front arrival) at each *θ*. The impulse delivered to the sphere is obtained by the time integration around a fully embedded fragment of mass *m* and density ρ_0 :

$$mv = \int F dt = \int_0^\infty dt \int_0^\theta 2\pi R_S^2 \sin\theta\cos\theta \cdot p(\theta,t)d\theta \tag{3}$$

and the final diffraction phase velocity is therefore:

$$v_{dif} = \frac{2\pi R_S^2 P_{CJ}}{3m \cdot (\frac{1}{\tau_1} + \frac{1}{\tau_2})} \tag{4}$$

Here τ_1 is proportional to the fragment placement radius r_S, and $\tau_2 \propto R_S / c$, where c is the sound velocity of the detonation products, and R_S is the spherical fragment radius. P_{CJ} is the CJ detonation pressure.

For external placement we took a similar approach. The detonation pressure is replaced by $\rho \cdot u^2$ at the air-products interface, ignoring the small acceleration due to the shock-compressed air. ρ and u are the density and the velocity of the detonation products. Denoting by Q the product $Q = \rho \cdot u^2 / c$, and using Autodyn simulations, an approximation $Q(r_C) = 2.3[GPa/(km/s)] \cdot \exp[-(r_C / R_C - 1)]$ was found. Thus, the velocity imparted in this case is obtained by substituting Q for P_{CJ}, and κ_1, κ_2 for τ_1, τ_2 in Eqs. (3,4), where $\kappa_1 \propto r_C$ and $\kappa_2 \propto R_S$. Here r_C denotes the radius of the expanding detonation products, and R_C is the initial charge radius. For transitional placement, i.e., for a partly embedded sphere, we perform the *θ*-integration separately for the inner and the outer domain, with the surface pressure distribution in the respective domain taken as outlined above.

Turning to the drag phase, we need an approximate model for the expanding detonation products flow field. We do so by invoking integral mass and energy conservation laws, using information about velocity and density distributions obtained from Autodyn simulations. Thus, the velocity distribution is taken as linear at all times. Defining $\xi = r/r_C$ and assuming that the charge-air interface is at constant pressure $P_f > Pamb$ ($Pamb = 1bar$ is the ambient pressure) the detonation products expand with the density distribution:

$$0 < t < t_1 \qquad \rho(\xi) = \begin{cases} \rho_0 & \xi < \xi_1 \\ \beta_f \rho_0 \dfrac{1-\xi}{1-\xi_1} & \xi_1 < \xi < 1 \end{cases} \tag{5}$$

$$t_1 < t \qquad \rho(\xi) = \beta_f \rho_1 (1-\xi) \qquad 0 < \xi < 1$$

Where t_1 is the earliest moment for which $\xi_1 = 0$, and β_f is a constant parameter related to P_f. The time $t = 0$ is taken here at the completion of the detonation. Conservation of mass requires:

$$\int_0^{r_c} 4\pi r^2 \rho(r,t)dr = \frac{4\pi}{3}\rho_0 R_C^3 = \frac{4\pi\rho_0}{12} r_C^3 [1 + \xi_1 + \xi_1^2 + \xi_1^3 + \beta_f (\xi - \xi_1 - \xi_1^2 - \xi_1^3)] \tag{6}$$

This equation can be solved for ξ_1 as a function of r_C. We then get

$$\xi_1 = \begin{cases} 1 & (R_C / r_C) = 1 \\ 0 & (R_C / r_C) \le [(1+3\beta_f)/4]^{1/3} \\ 0 < \xi_1 < 1 & [(1+3\beta_f)/4]^{1/3} < (R_C / r_C) < 1 \end{cases} \tag{7}$$

When $\xi_1 = 0$, ρ_0 is replaced by ρ_1 and we assume $\rho_f = \beta_f \rho_1$, the equation for ρ_1 is (after calculating the total explosive mass at this state):

$$\rho_1 = \rho_0 (\frac{R_C}{r_C})^3 (\frac{4}{1+3\beta_f}) \tag{8}$$

The kinetic energy of the detonation products is:

$$E_K = \int_0^{r_c} 4\pi r^2 \rho(r,t) \frac{1}{2}[u(r,t)]^2 dr = \quad (9)$$
$$= \frac{\pi}{15}\rho_0 r_C^3 u_C^2 [1 + 5\beta_f + (1-\beta_f)(\xi_1 + \xi_1^2 + \xi_1^3 + \xi_1^4 + \xi_1^5)] \qquad 0 < \xi_1 < 1$$

We assume that the kinetic energy of the detonation products decreases by the amount of work performed in compressing the ambient air, where the latter is estimated as proportional to the volume of the expanding products. The products-air interface velocity u_C is then calculated from Eq. 9 as function of r_C, which is updated in time according to $dr_C = u_C dt$. Assuming a constant value of the drag coefficient C_D, the drag force is then given by:

$$F = 0.5 C_D \rho(r_C) \cdot [u(r_C) - \dot{r}_S]^2 \quad (10)$$

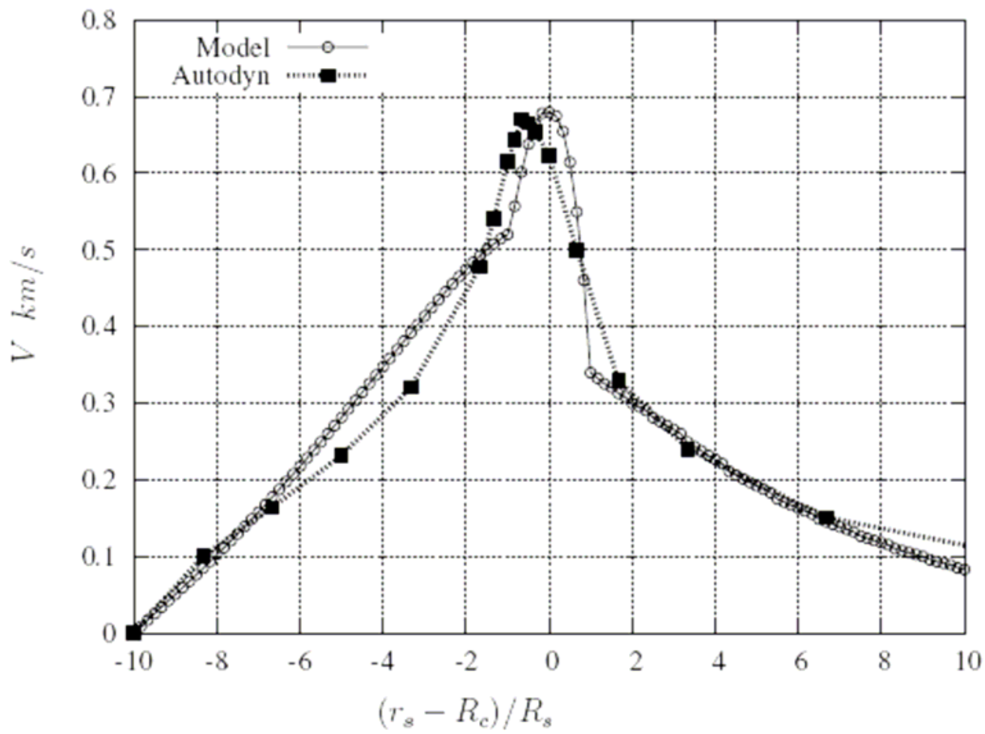

Figure 5. Final velocity (km/s) of a sphere embedded in or out of the charge, compared to Autodyn simulation

The density ρ is calculated according to the distribution (5), where $\xi = r_S / r_C$, and $u = \xi \cdot u_C$. The final velocity is then obtained by integrating the sphere acceleration $\ddot{r}_S = F / m$, along with the relation for the sphere location $dr_S = \dot{r}_S dt$. A comparison of this model prediction with Autodyn simulations is shown in Fig. 5. Generally speaking, the agreement between the two is quite good, especially for fully internal or fully external placement. For the transition zone, the model predicts a velocity peak for a slightly internal sphere center placement. The model clearly demonstrates the significant effect of the reversed acceleration due to wave diffraction around the spherical fragment, which is the main cause for the internal-placement velocity peak observed in Held's experiments.

SUMMARY

A simple gas-dynamic model for the acceleration of a fragment by explosive is proposed, with results that agree with Held's experimental data, as well as with numerical simulations. The model assumes that the fragment, initially placed at an arbitrary point relative to the charge face, begins to move when engulfed by the detonation front or the expanding explosive products. The total velocity increment on a spherical fragment positioned in air or within the explosive may be regarded as consisting of two parts: An abrupt "dynamic force" due to wave diffraction around the fragment, and a continuous "drag force" due to entrainment by the oncoming flow.

Under this model, we neglect the change in the gas flow-field due to interaction with the fragment. This approximation is valid as long as the fragment mass is much smaller than the charge mass.

While Gurney's model is aimed solely at the final velocity, we consider the time evolution of the products gas flow along with the gradual acceleration of the fragment.

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