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RICOCHET OF DEFORMING PROJECTILES FROM DEFORMING PLATES

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Summary—Ricochet means rebound of a striker from the impacted surface (or penetration into a medium along a curved trajectory emerging through the impacted surface with a residual velocity). Changes in direction, velocity and rotational motion of the penetrator are due to several mechanisms. These include release of stored elastic impact energy; influence of surfaces, material interfaces and impact deformations in the target on the magnitude and direction of the resisting force during impact; resistance to motion due to drag and friction. The subject is of interest due to the need to establish safety zones and to design containment structures to guard against failure of rapidly moving machine parts, to protect outer components of space vehicles from the energetic debris spray resulting from oblique hypervelocity impact and to reconstruct bullet trajectories in forensic engineering. This paper contrasts two-dimensional plane strain calculations of ricochet with fully three-dimensional simulations performed with *Apollo*, a three-dimensional Lagrangian finite element code for impact and explosive loading problems set up exclusively on personal computers and workstations. While some useful information can be extracted from plane strain calculations regarding the early stages of impact, the use of two-dimensional calculations to simulate fully three-dimensional phenomena with long response times (up to the millisecond regime) results in gross overestimation of deflections and is inherently dangerous. Copyright © 1996 Published by Elsevier Science Ltd.

Keywords: ricochet, penetration, high obliquity impact, forensic engineering.

BACKGROUND

When a striker rebounds from the impacted surface of a target, or penetrates along a curved trajectory emerging through the impacted surface with a residual velocity, its behavior is defined as ricochet. Three major factors affect changes in velocity and direction associated with ricochet:

(a) impact pressures compress and deform striker and target. The subsequent recovery of the stored elastic energy results in motion changes.

(b) the characteristics of the target (surface properties and geometry; material properties; material interfaces) and its subsequent deformation after impact govern the direction and magnitude of the resisting force resultant which acts on the striker.

(c) resistance to motion due to drag and friction reduces velocities.

The component of the resisting force resultant which is aligned with the direction of motion slows the penetrator, which may be a "drag" force. The component of the resultant which acts normal to the direction of motion causes direction change and may be thought of as the "lift" force. If the resisting force resultant does not act on a line which intersects the center of gravity, the striker experiences rotating and bending moments [1–2].

Ricochet is of interest for a number of reasons. First, there is interest in the basic mechanics of ricochet for rigid and deformable media. Among these is the ingenious work of Johnson, Sengupta and Ghosh [3–4] who fired long rods against relatively thick plates, both of Plasticine (modeling clay), at obliquities ranging from 0 to $<75°$ (obliquity is the impact angle measured from the normal to the plate surface). They were able to ascertain the mechanics of target cratering and define conditions leading to projectile breakup. Many of their observations have since been verified computationally and experimentally for metallic materials. Tate [5] derived an expression for the ricochet angle (also measured from the plate normal) for long rods from thick plates which agreed well with the observations of Johnson, Sengupta and Ghosh. An extensive experimental program involving ricochet of metallic spheres from metallic targets was undertaken by Backman and Finnegan [6]. Considerable data on ricochet from sand, clay, water and concrete can be found in the reports by Recht and his colleagues, cited above, as well as Hutchings [7], Birkhoff [8], Bushkovitch [9], Kemper and Jones [10], Johnson and Daneshi [11–12], Daneshi and Johnson [13] and many others.

The safety of personnel and equipment threatened by ricocheting projectiles or fragments has been an ongoing concern. Typical of such studies are the works of Dunn and Dotson [14], Hayes *et al.* [15] and Reches [16].

Current interests in ricochet include the areas of hypervelocity impact and forensic engineering. Much of the work in hypervelocity impact involves normal incidence. However, most impacts in reality occur at obliquity. Many space structures in use or in the design phase have irregular outer surfaces or protrusions which are vulnerable to the debris clouds that can be generated by the breakup of projectiles impinging at high obliquity. Schonberg and Taylor [17–18] and Schonberg [19] have experimentally investigated oblique hypervelocity impact and ricochet for aluminum dual-wall structures. They developed a set of empirical equations that characterize observed penetration phenomena as a function of the geometric and material properties of the impacted structure and the diameter, obliquity and velocity of the impacting projectile. Burke and Rowe [20] review bullet ricochet from the standpoint of forensics. A knowledge of the wounds suffered by shooting victims, the deformation of bullets or shotgun pellets and ricochet marks on surfaces at the scene of a crime can be instrumental in reconstruction of the shooting.

NUMERICAL SIMULATION OF RICOCHET

Ricochet is a fully three-dimensional phenomenon. There are pitifully few analytical models which treat oblique incidence. These tend to be either extremely simple or limited to the initial stages of the impact process. Not uncommonly, some empiricism is involved in model development. The ricochet problem can therefore be treated experimentally or with finite element or finite difference codes as a fully three-dimensional solution or with the plane strain approximation.

Experiments take some time to set up, but once materials have been fabricated and the experimental arrangement is in place, 1–4 shots daily may be obtained, depending on whether the experiments are performed at full scale or model scale. Experiments are costly. A typical cost for a single full-scale shot is upwards of $10,000, depending on the complexity of the experimental arrangement, the degree of instrumentation and the data reduction required. Model scale tests in enclosed ranges can be done for about $2,000 per shot. Note that these costs are per shot, nor per data point. Frequently, because of excessive projectile yaw, malfunction of instrumentation or other causes, several shots may be required to obtain a valid data point. Even then, the information extracted from a ballistic test is minimal from the point of view of an analyst—initial and final velocity and orientation of the projectile, residual projectile mass, target deformation and mass loss. Inevitably there is scatter in the data due to variations in material properties of nominally identical materials and uncertainties in initial and boundary conditions. Time and cost constraints almost never permit acquisition of a data base with enough variation of parameters to construct unambiguous analytical models.

The cost of three-dimensional simulations is also high, even on supercomputers [21]. The exact cost depends on the code and computer used, the spatial and temporal resolution, the number of sliding interfaces in the calculation and the constitutive model. However, the cost of a three-dimensional calculation on a supercomputer almost always exceeds the cost of an equivalent experiment. The advantage of the computations is the quantity of information obtained: full time-resolved displacements, strains, strain rates, momenta, energies, forces, moments and so on. Coupled with experimental data, this forms an excellent base for construction of approximate analytical models for parametric studies. The validity of computational results depends on the constitutive model employed, the source of data for the parameters of the constitutive model and the degree of material failure in the experiment and how successfully it is simulated computationally.

In order to avoid the high cost of computing in three dimensions, recourse is sometimes made to two-dimensional calculations employing the plane strain assumption. Two-dimensional plane strain calculations are straightforward enough, relatively inexpensive and provide some useful information about the early stages of impact. However, when the oblique impact of a striker is treated as the impact of an infinitely long wedge [see Fig. 1], important physical phenomena are being neglected, not the least of which are the out-of-plane motions leading to lateral stress relaxations. Useful qualitative information (and, if care is taken with the calculations, useful quantitative information) may be obtained from plane strain solutions for the early stages of an oblique impact. Their utility, however, degrades with increasing time after impact so that for late times, when important aspects of

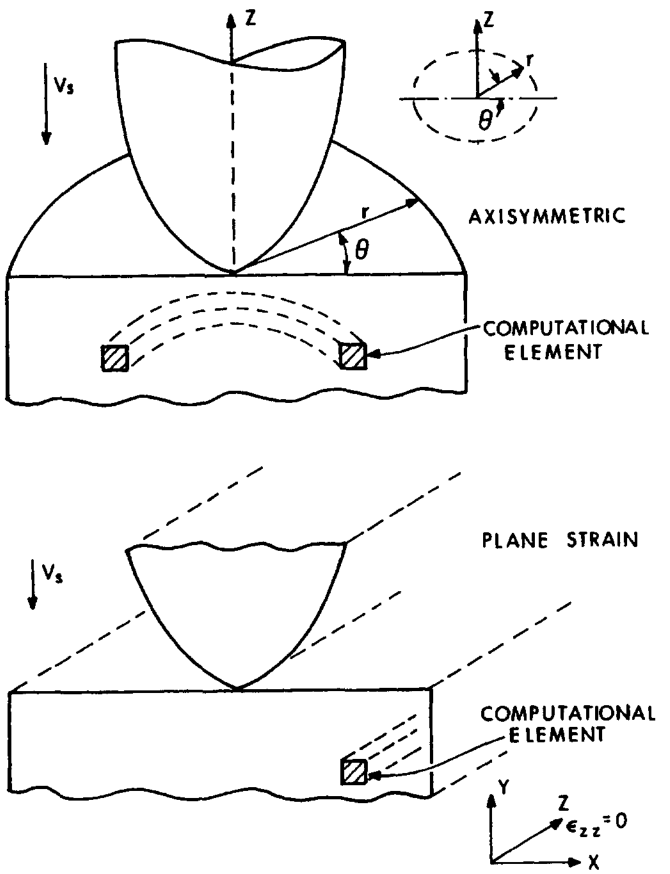

Fig. 1. Computational elements for axisymmetric and plane strain calculations.

penetration and target response (such as bending and shear failure) are being determined, plane strain solutions are speculative at best.

The nature of the plane strain approximation has been discussed by many authors [22–25]. The general conclusion of these studies is that, with appropriate scaling, useful information regarding overall kinetic energies, momenta and velocities may be obtained. However, after the time that a release wave would have returned from the lateral boundary of an actual three-dimensional calculation, plane strain calculations are not useful for extracting information related to the internal energy of the problem. They require less energy for deformation than their exact (2D axisymmetric or 3D oblique impact) counterparts and grossly overestimate late-time deformations.

If they are so fraught with risk, then, why perform plane strain calculations at all? Two reason stand out: they are cheap and, occasionally, they produce useful results. Norris *et al.* [26] used the HEMP code (existing production wave propagation codes are reviewed in detail in chapter 9 in Ref. [27]) in plane strain mode to supplement information from model scale oblique impact experiments of long rods striking thin plates at high obliquity and high velocity in order to determine optimum material and geometry configurations for high length-to-diameter (L/D) ratio projectiles. They obtained excellent agreement between calculations and experiments, primarily because penetration was achieved at about the time the reflected wave from the penetrator's lateral dimension arrived. Zukas and Segletes [28] showed close correlation between experiments and calculations for hypervelocity projectiles striking spaced thin-plate targets at extreme obliquity for deformations and debris spray angles. Other simulations have been less successful. Jonas and Zukas [29] modeled long rod ricochet with the EPIC code. Good agreement between plane strain calculations and computed rod deformations was obtained early on after impact. However, comparison of calculations with radiographs at late times after impact showed poor correlation. The plane strain calculations severely underestimated rod bending. Detailed comparisons of the progressive deviation of plane strain calculations from exact (axisymmetric) calculations of long rod impact using the HELP code are shown by Zukas, Jonas and Misey [23].

Provided some care is taken in setting up calculations, and considerable skepticism employed in interpreting results, plane strain calculations can provide some insights into three-dimensional behavior. For highly energetic problems, the early-time response can be predicted quite well. However, it would be both foolish and dangerous to rely on plane strain calculations alone for design without corroboration by experiments or exact analyses.

COMPUTATIONAL RESULTS

1. *The* ZeuS *and* Apollo *codes*

Several calculations were performed to test the newly-developed *Apollo* code, a three-dimensional (3D) finite element Lagrangian code implemented on personal computers and workstations. The same calculations were performed in plane strain using *ZeuS*, a two-dimensional (2D) finite element code [30–37]. Results were compared with experiments performed by Ipson *et al.* [2].

Since the advent of hydrocodes in the early 1960s, a rich collection of algorithmic information has been accumulated for both Lagrangian and Eulerian calculations employing finite elements or finite differences. Hence it is much easier today to develop a computational tool than it was some 30 years ago when code developers were also pioneers. The development of *Apollo* made heavy use of existing algorithmic information, drawing very heavily on the work of Belytschko and his associates.

Apollo is an explicit time integration program for the nonlinear analysis of solids and fluids subjected to fast, transient loading. At present, the only element available is an isoparametric hexahedral element consisting of eight nodes, 24 degrees of freedom and six sides. A Lagrangian mesh description is employed, making use of Cauchy stresses and velocity strains together with the

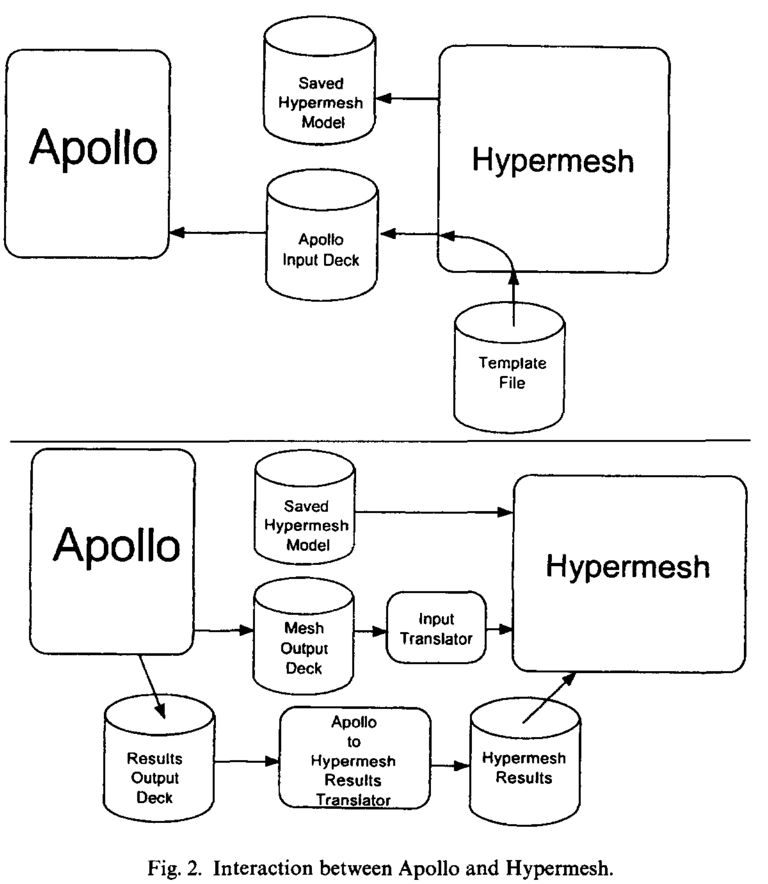

Fig. 2. Interaction between Apollo and Hypermesh.

Table 1.

Material properties		
	Projectile	Target
Density (gm/cc)	7.85	7.85
Shear modulus (dynes/cm**2)	2.66E + 07	2.66E + 07
Strength (dynes/cm**2) (yield & ultimate)	2.00E + 10	1.14E + 10
Strain at failure	1.2	1.6

30 Degree problem

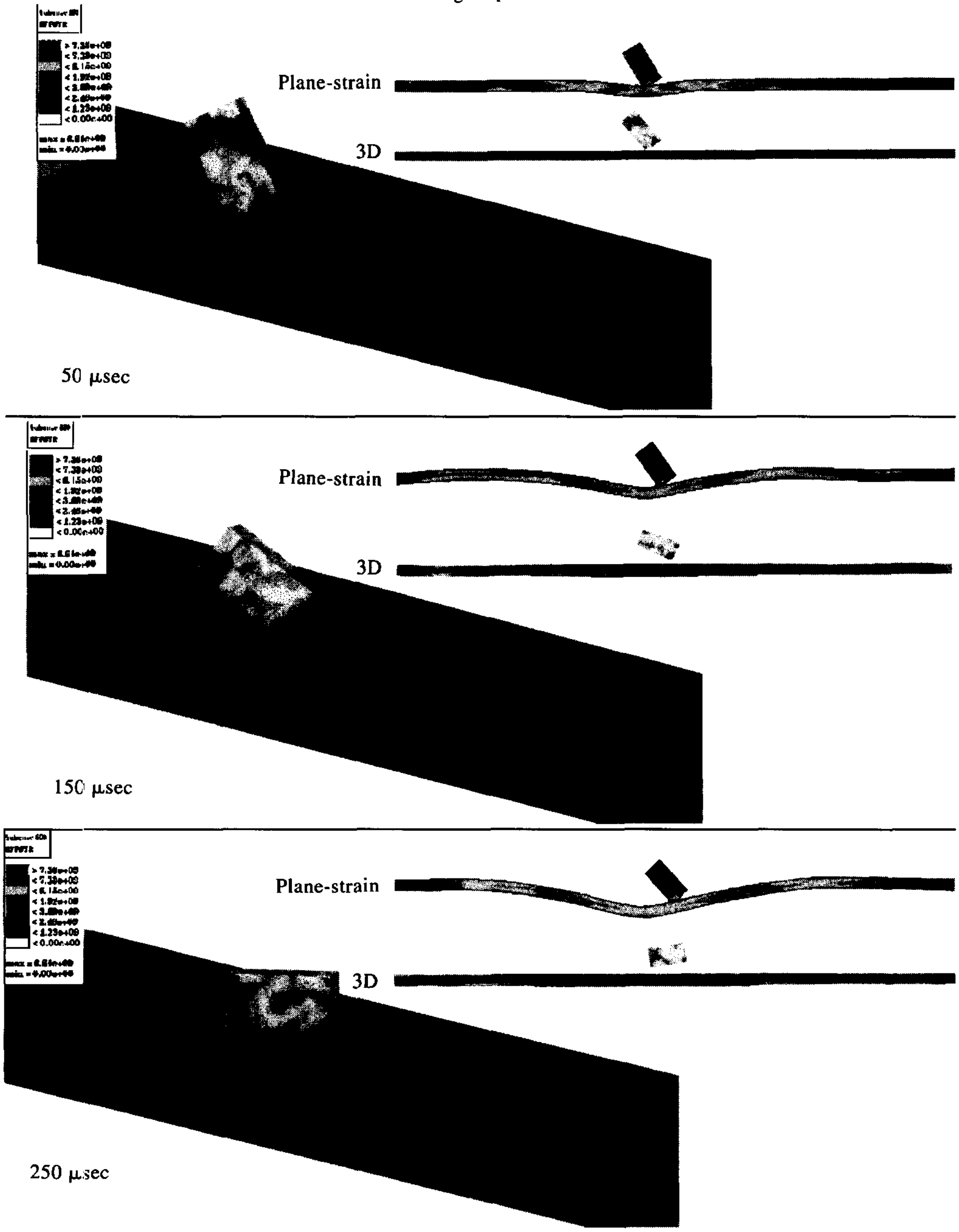

Fig. 3. Deformations at various times after impact—30° problem.

Jaumann rotation rate. One quadrature point is used to evaluate stress and strain fields in the element. This implies that for evaluation of nodal forces, the stresses and strain rates are constant. The assumption of a constant stress field means that certain deformation modes of the element are not resisted by nodal forces. This phenomenon is known as hourglassing [38]. To avoid the severe mesh distortion brought about by hourglassing, a procedure developed by Flanagan and Belytschko [39] is used.

Apollo can support multiple materials in a single problem. Material strength is modeled with an elastic-viscoplastic hardening model. The von Mises yield criterion is used to detect the onset of plasticity. High-pressure response of materials is computed using the Mie-Gruneisen equation of state. An erosion criterion may be specified to remove excessively distorted elements from the calculation. When erosion occurs, new sliding surfaces are computed dynamically. This initial version of *Apollo* allows only a single sliding interface.

The Pinball algorithm [40] is used for contact–impact. The main concept of the Pinball algorithm is to enforce the impenetrability condition and contact conditions on a set of spheres, or pinballs,

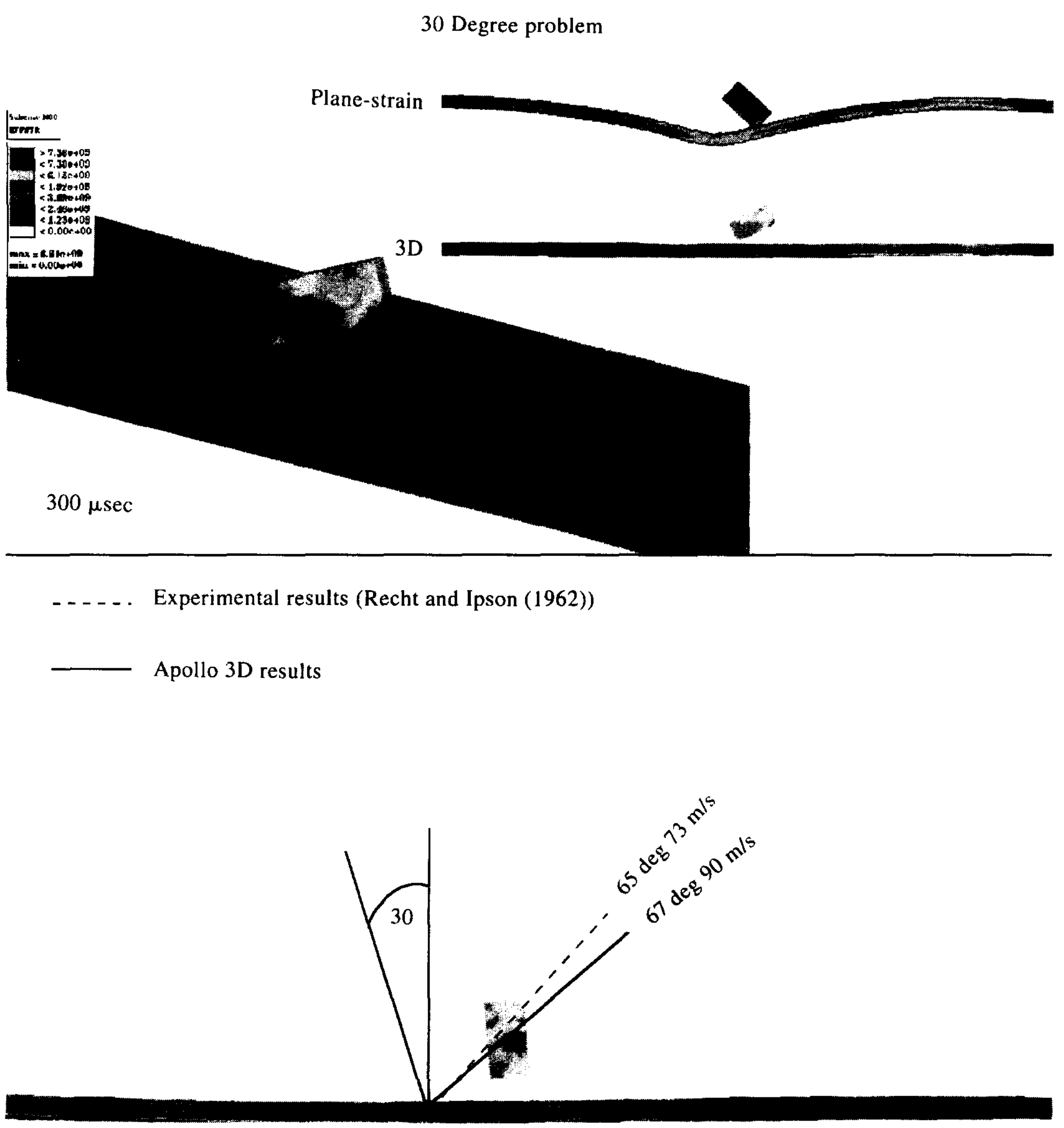

Fig. 4. Comparison of experimental and computational results.

which are embedded in the finite elements. In the first cycle, the radius needed to create a sphere of volume equal to the element volume is computed for each element. For elastic-plastic problems most of the element deformations can be considered nearly incompressible. Therefore, the element volume, and also the radii of the pinballs, will change little over the course of a simulation. For this reason, the radii are calculated only once. For penetration detection on subsequent cycles, the distance between centers of each slave pinball and each master pinball are calculated and compared with the sum of the radii of the two elements. If the distance is less than that sum, penetration has occurred and a corrective force is applied. (See Refs [40–41] for details and applications).

The Pinball algorithm greatly reduces the time required in the search for penetration over conventional algorithms using put-back logic since interpenetrability becomes a simple check of distance between two pinballs and, since it involves almost no recursive calculations, it lends itself readily to vectorization. We have found that it works very well for impact calculations that are finely meshed in the vicinity of the contact point. If the element size is too large, if element aspect ratios greatly exceed one, or in low-speed impacts where the time of contact is crucial, then the pinball algorithm can be less than ideal. Because penetration by hexagonal elements is treated as penetration by spheres, penetration detection may be delayed at element corners and be premature at element faces. However, these details are insignificant compared to the time savings for high speed problems with apropriate meshes.

Pre- and post-processing for *Apollo* are done through the separate commercial package *HyperMesh* [42–44]. *HyperMesh* is a well-integrated finite element analysis (FEA) model design package that can be extremely flexible, depending on the application. It can serve as a post-processor as well. However, the underlying design of *HyperMesh* assumes that element connectivity will not change in the course of a calculation. This is true for Lagrangian calculations where deformations are limited. However, for large deformation problems experiencing material failure where element connectivity can change throughout the problem, many of the post-processing features of *HyperMesh* are lost. This applies to codes with erosion or element advection capabilities such as LS-DYNA, *ZeuS*, *Apollo*, PRONTO, DYSMAS-L, AUTODYN, EPIC and others under development.

A template file is read by *HyperMesh* and used to specify input–output characteristics between *Apollo* and *HyperMesh* [see Fig. 2]. The model is also saved in the *HyperMesh* native format. *Apollo* reads the resulting input deck generated by *HyperMesh*, performs the calculations to the specified problem time and creates a results output deck containing stresses, strains, pressures, velocities and other variables of interest. If connectivity changes occur in the course of a calculation a mesh output deck containing node and element information is generated as well. The results output deck is then translated into a results file native to *HyperMesh*. The mesh output information, if needed, can be loaded directly into *HyperMesh* by an input translator called by the HyperMesh application. Using

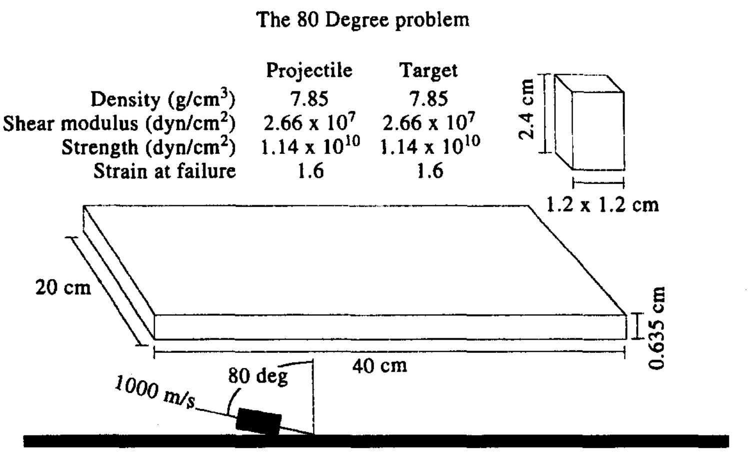

Fig. 5. Problem setup for 80° impact.

this informtion, the analyst can obtain from *HyperMesh* contour plotting, animation, displacement plots, cutaway views, rotations (depending on platform) and many other valuable aids to understanding the computational results.

2. *Calculations*

Several calculations were performed with *ZeuS* and *Apollo* to illustrate the differences between two-dimensional plane strain and fully three-dimensional calculations involving the oblique impact

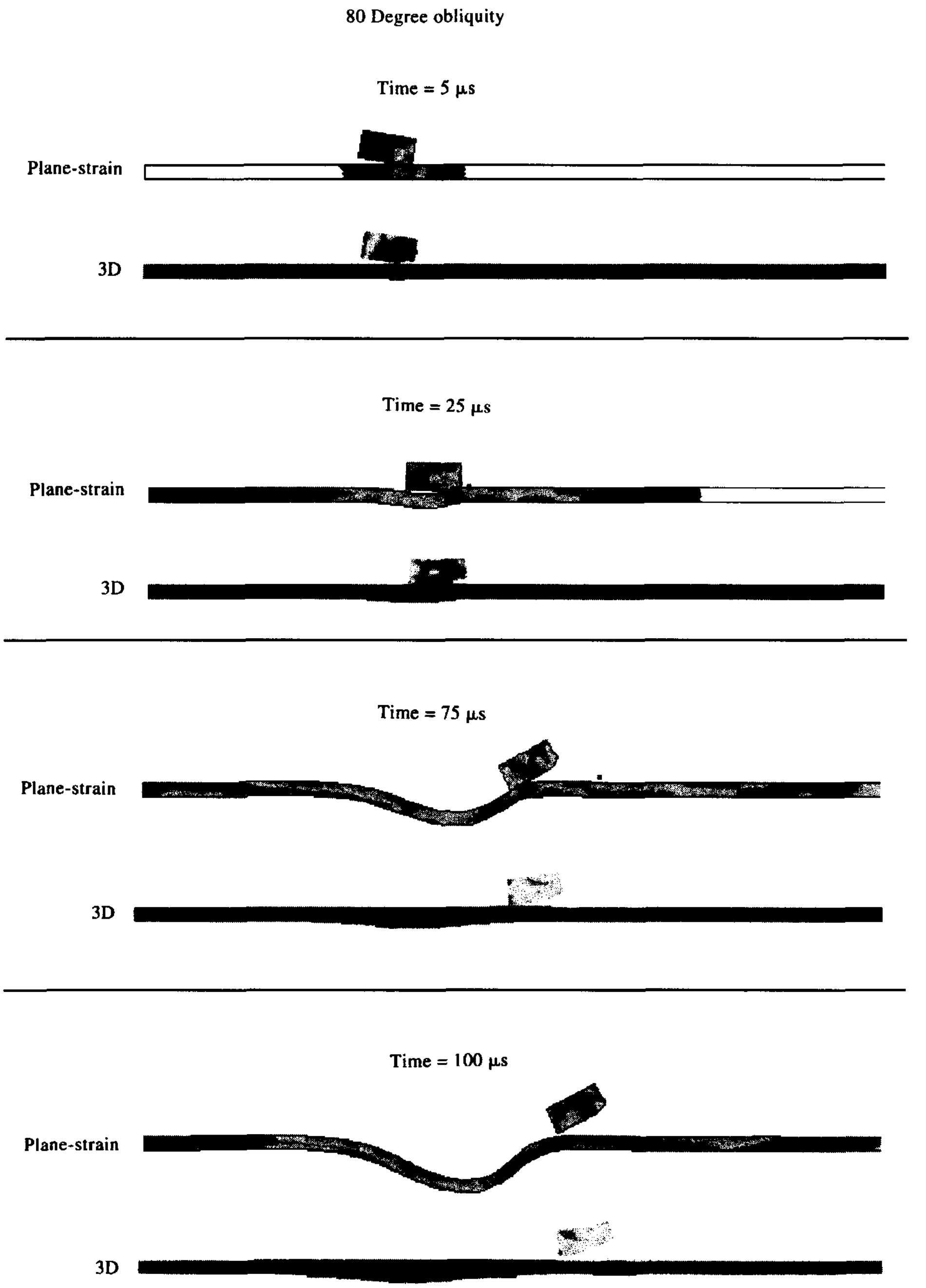

Fig. 6. Deformations at various times after impact—80° problem.

and ricochet of compact projectiles. The first involved the impact of a hardened steel (Rockwell C 53) 1.27 cm rectangular cross-sectional projectile with L/D = 2 and a mass of 26.5 g at an obliquity of 30° and velocity of 180 m/s against a 4130 steel plate with a thickness of 0.635 cm. Material properties for the calculations are given in Table 1. Equation of state data for steel was taken from Ref. [45].

Figure 3 shows deformation profiles of the 2D and 3D calculations at three different times. At each time, an isometric view of the 3D calculation is presented which also shows surface stress contours. In addition, a cross-sectional view of the 2D plane strain calculation is shown and contrasted with an edge view of the 3D calculation. Note the progressive deviation of the 2D plane strain results from the 3D calculation with time. For this problem, the experimentally-determined ricochet angle was about 65°. (There is considerable scatter in the data [1].) The ricochet velocity was 73 m/s. The *Apollo* results indicate a ricochet angle of 67° and a velocity of 90 m/s (Fig. 4). No attempt was made to get similar information from the 2D calculation because of its obvious diversion from 3D results and reality.

Both calculations were run on a Silicon Graphics Indigo Extreme workstation. The 2D calculation employed 1,593 nodes and 2,736 elements and ran for a little over 68 min, resulting in a whiz factor of 8.6×10^{-5} CPU seconds/node/cycle. The 3D computational grid consisted of 21,241 nodes and 15,432 elements. The total run time was 7.6 hr, giving a whiz factor of 1.3×10^{-4} CPU seconds/nodes/cycle.

A second set of calculations were performed using the same geometry and materials but changing the obliquity to 80° and the striking velocity to 1 km/s. The initial configuration is shown in Fig. 5. Computational results are shown in Fig. 6. Note that in this case, where the normal component of velocity is 174 m/s, less than one-fifth of the striking velocity, the 2D plane strain calculations still overestimate target deformation, which, in reality, for both cases was minimal, but otherwise mimics the three-dimensional results. The ricochet angle predicted from the 2D plane strain and 3D calculations was 68 and 72°, respectively. The same spatial resolution was used in the 80° problem as in the 30° problem. Total run time for the plane strain calculation was 110 s while the 3D calculation required 28 min to reach a problem time of 100 μs.

CONCLUSIONS

Plane strain calculations, if used with considerable caution, can produce results which have some qualitative value. However, because of the inherent difference in the physics modeled in the 2D plane strain and 3D calculations, good correlation between the two should not be expected. Plane strain results should never be used for design purposes unless corroborated by exact analyses, experiments or both.

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