# POST-PERFORATION LENGTH AND VELOCITY OF KE PROJECTILES WITH SINGLE OBLIQUE TARGETS 

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#### Abstract

In the context of vulnerability studies there is a strong need for simple methods for predicting the perforation limit and - in case of perforation the remaining length and velocity. An empirical formula for the perforation limit of long rod penetrators with single oblique targets has been deduced from a non-dimensional ansatz with analytical constraints. The following magnitudes enter the formula: material properties (tensile strength and density), length to diameter ratio of penetrator, obliquity and thickness of target and impact velocity. If the impact velocity is greater then the one required for the perforation limit, the remaining length and velocity of the penetrator can be determined with a simple calculation on the base of the penetration formula.


## INTRODUCTION

At the 13th International Symposium on Ballistics an ansatz has been presented for calculating the perforation limit of APDSFS ammunition in the caliber range of 105 to 140 mm hitting an oblique single target [1]. In the meantime this ansatz could be refined by including results about the influence of tensile strength of the target material. The necessary experiments were carried out in our indoors firing facility. The set of experiments now available covers the following range of values:

- 74 test results with 19 different penetrators
- Calibers 25, 30, 35, 105, 120, 140 mm
- Penetrators properties

| Lengths | L | $90-825$ | mm |
| :--- | :--- | ---: | :--- |
| Diameters | D | $8-32$ | mm |
| Length to diameter ratios | $\mathrm{L} / \mathrm{D}$ | $11-31$ |  |
| Rod densties | $\rho_{\mathrm{P}}$ | $17000-17750$ | $\mathrm{~kg} / \mathrm{m}^{3}$ |
| Rod masses | $\mathrm{m}_{\mathrm{P}}$ | $0.1-0$ | kg |
| Impact velocities | $\mathrm{v}_{\mathrm{T}}$ | $1100-1900$ | $\mathrm{~m} / \mathrm{s}$ |

- Target properties

| Plate thickness | d | $40-400$ | mm |
| :--- | :--- | ---: | :--- |
| Tensile strength | $\mathrm{R}_{\mathrm{m}}$ | $800-1600$ | MPa |
| Obliquities (NATO) | $\theta$ | $0-74$ | $\circ$ |
| Density | $\rho_{\mathrm{T}}$ | 7850 | $\mathrm{~kg} / \mathrm{m}^{3}$ |



Fig 1: Definitions

The effective penetrator length L is defined in the following way: starting from the actual penetrator the tip is replaced by a cylinder of equal mass and diameter D and the remaining length reduced by D (see Fig 1).

## PERFORATION LIMIT

The perforation formula is composed of four dimensionless terms with separate representation of the influences of length to diameter ratio $\mathbf{T}_{\mathbf{1}}$, target obliquity $\mathbf{T}_{\mathbf{2}}$, density ratio of penetrator to target $\mathbf{T}_{3}$ as well as material properties and incident velocity $\mathbf{T}_{4}$. The formula is valid for $\mathrm{L} / \mathrm{D}$ greater than 10 and within the range of experimental values as listed above.

$$
\begin{equation*}
\frac{d}{D}=a\left(\frac{L}{D}\right) \cdot(\cos \theta)^{m} \cdot \sqrt{\frac{\rho_{P}}{\rho_{T}}} \cdot e^{\frac{-c \cdot R_{m}}{\rho_{P} \cdot v_{T}^{2}}} \tag{1}
\end{equation*}
$$

$\begin{array}{llll}T_{1} & T_{2} & T_{3} & T_{4}\end{array}$
where: $\quad a\left(\frac{L}{D}\right)=\left(\frac{L}{D}\right)+3.94 \cdot\left(1-\tanh \frac{\frac{L}{D}-10}{11.2}\right)$

$$
\mathrm{c}=22.1+1.274 \mathrm{e}^{-8} \quad \cdot \mathrm{R}_{\mathrm{m}}-9.47 \mathrm{e}^{-18} \quad \cdot \mathrm{R}_{\mathrm{m}}^{2}
$$

$$
\mathrm{R}_{\mathrm{m}} \text { measured in }[\mathrm{Pa}]
$$

$$
\mathrm{m}=0.775
$$

## Influence of length to diameter ratio

The term $\mathbf{T}_{1}$ tends to $\mathrm{L} / \mathrm{D}$ as L/D gets large (L/D greater than 20). The plate thickness for limiting perforation is given by

$$
\begin{equation*}
\mathrm{d}=\mathrm{L} \cdot(\cos \theta)^{\mathrm{m}} \cdot \sqrt{\frac{\rho_{\mathrm{P}}}{\rho_{\mathrm{T}}}} \cdot \mathrm{e}^{\frac{-\mathrm{c} \cdot \mathrm{R}_{\mathrm{m}}}{\rho_{\mathrm{p}} \cdot v_{\mathrm{T}}^{2}}} \tag{4}
\end{equation*}
$$

## Considerations for the transition region

For perpendicular impact (obliquity $=0^{\circ}$ ) and very high velocities, the terms $\mathrm{T}_{2}$ and TS tend to 1 . For $\mathrm{L} / \mathrm{D}$ greater than 20 the penetration formula now becomes identical to the one for hydro-dynamic penetration:

$$
\begin{equation*}
\mathrm{d}=\mathrm{L} \cdot \sqrt{\frac{\rho_{\mathrm{P}}}{\rho_{\mathrm{T}}}} \tag{5}
\end{equation*}
$$

## Influence of tensile strength of the tar get plate

The product $\mathrm{c}-\mathrm{R}_{\mathrm{m}}$ is increasing with increasing tensile strength up to 1300 MPa and then remains practically constant up to the the investigated tensile strength of slightly more than 1600 MPa . Thus an increase of target tensile strength beyond 1300 MPa did not result in a decrease of the limiting perforation length in our experiments. If this behaviour is valid in general cannot determined, because in this range of tensile strength, to date we have only few results available as can be seen from Fig 2. We intend to carry out additional experiments. However with the penetration formula presented here the range of 700 to 1300 MPa is covered reliably.


Fig 2: Values for $\mathrm{c}-\mathrm{R}_{\mathrm{m}}$ from experimental results

## Accuracy of the formula

In Fig 3 the results of 74 perforation limits are presented in a dimensionless form. The axes have been chosen as follows:Fig 3: Accuracy of the formula

$$
\text { x-axis: } \quad \frac{c \cdot R_{m}}{\rho_{P} \cdot v_{T}} \quad y \text {-axis: } \quad \frac{d}{D} \cdot a\left(\frac{L}{D}\right)^{-1} \cdot(\cos \theta)^{-m} \cdot \sqrt{\frac{\rho_{T}}{\rho_{P}}}
$$



Fig 3: Accuracy of the formula

The correspondance between experiment and the formula is good. The maximum differences are only $6 \%$ and the standard deviation is $2.6 \%$.

## POSTPERFORATION LENGTH AND VELOCITY

## Terminal ballistc test procedure

A smooth bore 38 mm experimental gun is used to launch test rods at velocities $\mathrm{VT}=1200$ $1600 \mathrm{~m} / \mathrm{s}$. We use rods with length to diameter ratios of 14 and 20 which are supported by an aluminium sabot. Targets are set at obliquities of either 0 or inbetween 42 and 52 degrees NATO. At short distance before the target a sheet of paper is placed in order to determine the yaw of the projectile (the yaw angle must not exceed $1^{\circ}$ ). The postperforation parameters (residual length and velocity) are determined with two flash X-rays. The trigger for the first X-ray is placed on the target itself, the trigger for the second at 0.67 m behind. The whole test arrangement is depicted in Fig 4.


Fig 4: Test arrangement

Fig 5 shows the picture of a residual projectile 0.2 ms after hitting the target, as recorded by the first X-ray. Fig 6 shows the same projectile after another 0.476 ms , recorded by the second X-ray. The effective flight distance in between the two pictures is 0.54 m which gives a residual velocity of $1134 \mathrm{~m} / \mathrm{s}$.


Fig 5: $1^{\text {st }}$ X-ray picture
Fig 6: $2^{\text {n }}$ d X-ray picture

## Definition of the effective residual length

Residual length $\mathrm{LR}_{\mathrm{e}} \mathrm{ff}$ is determined with the help of X-ray pictures right behind the target. The total length of the residual projectile is composed of the length of the practically non-deformed part of the projectile and the average length of the head fragments (see Fig 7)


Fig 7: Definition of residual length

## Postperforation penetrator velocity

Postperforation velocity could be calculated in a simple manner. Fig 8 presents our experimental results and shows the dependence of scaled residual velocity on the ratio of actual perforation to perforation limit according to formula (1).


Fig 8: Test results together with approximation function.

The penetrator tail is gradually slowed down by the elastic shock waves which are induced by reflection off the penetrator tip. In a tension-free rod the velocity difference can be deduced with considerations from continuum mechanics:

$$
\begin{array}{llll} 
& \sigma_{P} & \text { tensile strength } \\
\Delta v=\frac{2 \cdot \sigma_{P}}{\sqrt{E_{P}} \cdot \rho_{P}} & \text { (6) } & \mathrm{E}_{\mathrm{P}} & \text { Young's modulus } \rightarrow \text { penetrator }  \tag{6}\\
& \rho_{P} & \text { density }
\end{array}
$$

For the present case a $\Delta \mathrm{v}$ of $45 \mathrm{~m} / \mathrm{s}$ yielded the best agreement with test results, demonstrated by the stair line which has been calculated on this base. If the tensile strength is determined from $\Delta \mathrm{v}$, the Young's modulus and the density, a value of 1800 MPa is obtained. In static pull experiments, a tensile strength of 1400 MPa has been measured. The difference can be explained by dynamic effects. Very good fitting results are obtained with the following ansatz:

$$
\begin{equation*}
\mathrm{v}_{\mathrm{R}} / \mathrm{v}=1+\mathrm{a} \cdot \ln \left(1-\mathrm{d} / \mathrm{d}_{\lim }\right) \tag{7}
\end{equation*}
$$

The coefficient a depends indirectly proportional on impact velocity and less strongly on material properties und geometry of the penetrator. Fitting of our test results gave a value of 0.14 . With consideration of the velocity level alone this value is already well suited for arbitrary estimates.

## Postperforation penetrator length

Fig 9 presents our test results of scaled residual length as a function of the ratio of actual perforation to perforation limit according to formula (1).


Fig. 9: Test results together with approximation function
The length $L$ is defined according to Fig 1. The residual length LR corresponds to the effective residual length $\mathrm{LR}_{\mathrm{e}} \mathrm{ff}$ according to Fig 7, reduced by one-and-a-half times the diameter D. With these definitions the following simple relation in between scaled residual length and scaled perforation is obtained:

$$
\begin{equation*}
L_{R} / L=l-(l-b)-d / d_{j i m}-b-\left(d / d_{j i m}\right)^{2} \tag{8}
\end{equation*}
$$

In the considered range of velocities of $1400-1500 \mathrm{~m} / \mathrm{s}$ the coefficient b takes the value of 0.2 . With higher velocities $b$ tends to zero i.e. the parabola is reduced to a straight line through points $(0 / 1)$ and $(1 / 0)$. With smaller velocities $b$ becomes larger than 0.2 , the parabola has $a$ slightly bigger curvature.

## CONCLUSIONS

Instead of complicated computing codes, the simple formulae presented here can be used effectively for parameter and performance studies and in vulnerability models.

## REFERENCES

[1] W. Lanz, W. Odermatt, "Penetration Limits of Conventional Large Caliber Anti Tank Guns / Kinetic Energy Projectiles", Proceedings of the 13th International Symposium on Ballistics (1992), Vol. 3, pp. 225-233.

