J. Phys. D: Appl. Phys., Vol. 12, 1979. Printed in Great Britain

A simple estimate of the minimum target obliquity required for the ricochet of a high speed long rod projectile

A Tate
Royal Armament Research and Development Establishment, Fort Halstead, Sevenoaks, Kent TN14 7BP

Received 11 May 1979, in final form 31 July 1979

Abstract. A simple model for the minimum obliquity required to induce ricochet of a high speed long rod projectile from a thick target plate is suggested.

1. Introduction

Although the problem of ricochet is of long standing, the impact conditions for which ricochet has been studied are usually very different from those discussed in this paper. In particular the conditions are such that the projectile is nondeforming and its shape is spherical rather than a long thin rod. Examples of this previous work together with more detailed references can be found in the papers of Richardson (1948), Johnson and Reid (1975), Backman and Finnegan (1976), Hutchings (1976), Soliman *et al* (1976), and Daneshi and Johnson (1977a,b, 1978).

In a general survey of the mechanics of penetration given by Backman and Goldsmith (1978) they define ricochet as meaning that the penetrator is deflected from the target without either being stopped or passing through the target. The angle between the projectile trajectory and the normal to the target plate is termed the angle of obliquity of the plate and for each impact velocity there exists a minimum angle of obliquity above which the projectile will ricochet.

The physical mechanisms operative during the impact of solid bodies are primarily determined by the striking velocity and possible classifications of the various velocity regimes have been given by Johnson (1972) and Jonas and Zukas (1978). With increasing impact velocity the stress levels increase and at a sufficiently high velocity both the rod and the target will deform in a hydrodynamic way in the vicinity of the interface. Moreover if the rod erosion rate exceeds the speed at which gross plastic distortion can propagate along the rod then, as pointed out by Tate (1977) and Recht (1978), all the gross deformation is constrained to occur very close to the interface within a region bounded by a surface which will be termed the deformation front. The effectively rigid rear of the rod is acted on by the dynamic compressive strength Y across the deformation front. The eroding rod material splays out from the interface and much of the kinetic energy is spent in displacing the surrounding target material, either forming a cavity on the side of the rod which makes an acute angle with the target surface or forming a jet on the other side. A good description of this process based on two-dimensional computer calculations has been given by Norris *et al* (1976).

0022-3727/79/111825+05 $01.00

As the rod moves more deeply into the target the material thrown up by the surface jet of the rod material eventually restricts the forward motion of the interface so rotating it to be more nearly normal to the rod axis. The asymmetric forces acting on the deformation front in the projectile during penetration will produce a couple on the effectively rigid rear tending to rotate the rod out of the plate. Provided these forces are intense enough and are maintained for a sufficient length of time the rotation may lead to ricochet. Neither of these conditions is likely to be met by a thin target plate and no attempt is made in this paper to estimate the effects of plate thickness.

2. Theory

Consider a rod striking an oblique plate as shown in figure 1. During the penetration process the rod is eroded in a complex manner whose detailed description could at

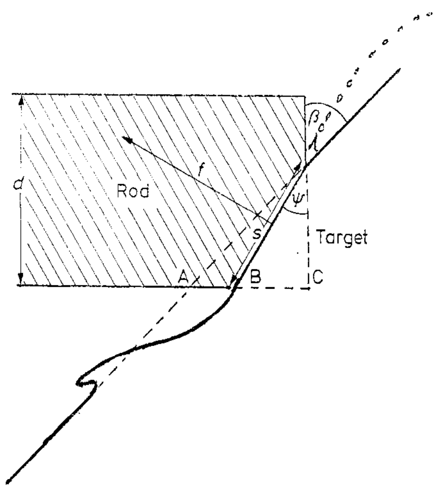

Figure 1. The rod–target configuration at time t after initial impact. AB$=Ut$, AC$=Vt$.

present only be given by a computer solution. A crude physical appreciation of the problem may be achieved however if we make the following approximations.

(i) The deceleration of the rod will be neglected.
(ii) The erosion of the rod surface will be taken to occur at the same rate as in the steady hydrodynamic phase.
(iii) The stress on the effectively rigid rear of the rod will be assumed to be Y_p, the strength factor in the modified hydrodynamic theory, which is closely related to the Hugoniot elastic limit of the material.
(iv) The cross-section of the rod will be taken to be a square. This considerably simplifies the calculations and is unlikely to alter significantly the qualitative picture of the ricochet condition that we are seeking.

If the impact velocity is V, the penetration velocity is U, Y_p and R_t are the projectile and target strengths and ρ_p and ρ_t are the projectile and target densities then the modified hydrodynamic approximation, as given by Tate (1967), shows

$$\tfrac{1}{2}\rho_p(V-U)^2+Y_p=\tfrac{1}{2}\rho_t U^2+R_t \qquad (1)$$

where for deep penetration $R_t \approx 3Y_t$. A target which is sufficiently strong will suffer no gross deformation unless the striking velocity V exceeds a critical value. If the target plate is oblique to the projectile trajectory and the critical velocity is not exceeded ricochet can occur. An estimate of the critical velocity may be obtained from equation (1) by setting U to zero giving

$$V_{\text{crit}} = [2(R_t - Y_p)/\rho_p]^{1/2}. \tag{2}$$

At higher velocities the projectile buries itself into the target but may also be rotated as described below.

Let the length of the rod be l and the side of square cross-section be d. Let us measure time from the instant of first striking the target and let the obliquity of impact be β. At an instant a short time t after initial impact the rod will approximate the idealised picture shown in figure 1. The length of eroding surface s is given approximately by

$$s = (V - U)\, t/\sin\psi. \tag{3}$$

The force f normal to the eroding surface is thus approximately

$$f = Y_p ds \tag{4}$$

and the moment c of this force about the centre of gravity of the rod is approximately given by

$$c = \tfrac{1}{2} fl \sin\psi \tag{5}$$

where

$$\tan\psi = \tan\beta(V - U)/V. \tag{6}$$

Thus from (3), (4) and (5)

$$c = \tfrac{1}{2} Y_p dl(V - U)\, t. \tag{7}$$

Assuming the rod is not deflected too much off course and certainly as a minimum estimate of the time t_m before the whole of the rod engages the target we have

$$t_m = d \tan\beta/V. \tag{8}$$

Now during this time the couple c will be inducing a rotational motion of the rod such that the yaw angle θ approximately satisfies the relation

$$\ddot{\theta} = 12c/M(l^2 + d^2) \tag{9}$$

where M is the mass of the rod and θ is the angular displacement. For a long rod the mass loss during the initial impact phase will usually be a small fraction of the total rod mass and, in conformity with the approximations of this simplified treatment, the rod mass will be taken as

$$M = \rho_p l d^2. \tag{10}$$

If at any stage the velocity of the end of the rod is directed out of the surface of the plate then it is postulated that the rod will ricochet. This can be expressed simply as follows:

ricochet if $\dot{\theta} > 2V \cot\beta/l$. (11)

Now using equations (7), (9) and (10)

$$\ddot{\theta} = At \tag{12}$$

where

$$A = 6Y_p(V-U)/\rho_p d(l^2+d^2). \tag{13}$$

Integrating (12)

$$\dot{\theta} = \tfrac{1}{2}At^2 \tag{14}$$

and using equations (8) and (11) ricochet would not be expected unless

$$\tfrac{1}{2}At_m{}^2 > 2V\cot\beta/l \tag{15}$$

or

$$Ald^2 > 4V^3\cot^3\beta \tag{16}$$

giving finally

$$\tan^3\beta > \frac{2}{3}\frac{\rho_p V^2}{Y_p}\left(\frac{l^2+d^2}{ld}\right)\left(\frac{V}{V-U}\right). \tag{17}$$

Using equation (1) it can be seen that for high impact velocities equation (17) tends to the form

$$\tan^3\beta > \frac{2}{3}\frac{\rho_p V^2}{Y_p}\left(\frac{l^2+d^2}{ld}\right)\left[1+\left(\frac{\rho_p}{\rho_t}\right)^{1/2}\right]. \tag{18}$$

It is clear from the above equation that the factors leading to a high obliquity before onset of ricochet are high rod-density, impact velocity and (l/d) ratio and low rod-strength factor. As an example of the magnitude of the ricochet angles to be expected consider a steel rod striking a steel target, the following physical parameters being assumed:

$\rho_p = \rho_t = 7{\cdot}8\times10^3$ kg m^{-3} $\quad Y_p = 2$ GPa $\quad V = 1{\cdot}5\times10^3$ m s^{-1}

l/d	β ricochet
5	75° 48′
10	78° 28′
15	79° 48′
20	80° 48′

As can be seen from the large obliquities shown in the table it appears to be very difficult to induce a high speed long rod to ricochet. Also, because the tangent of the critical angle of obliquity varies as the cube root of the parameters on the right of equation (18), the critical angle is relatively insensitive to variations of these parameters and is within a degree or two of 80° for the condition of present interest.

3. Conclusions

To reduce this complex interaction problem to the simple form given above has involved several substantial approximations. The deformation front surface is not known with any precision, it cannot be observed experimentally and computer solutions tend to smear out of the pressure distribution too much. It has been assumed that the deformation front remains at a constant angle to the rod axis until all the rod has engaged the target whereas some computer solutions would indicate that as the rod becomes deeply buried in the surface the deformation front becomes more nearly normal to the rod axis thus decreasing the applied turning moment. The effect of the rod surface erosion is to decrease both the moment arm of the offset force and the moment of inertia of the rod relative to the values given above. If the tangent of the ricochet angle is changed by 20% due to these effects this should lead to an average change of about 2° in the ricochet angle. Because of the high obliquities involved a target which is more than a rod diameter thick may be expected to behave as a thick target. The assumption of constant rod velocity is not too severe because the velocity drop during this phase is not expected to be more than one or two per cent. At present it would appear that any more detailed theoretical approach to this problem would most profitably be pursued through computer calculations.

References

Backman M E and Finnegan S A 1976 *US Naval Weapons Centre, China Lake, California. Tech Rep. TP 5844*
Backman M E and Goldsmith W 1978 *Int. J. Engng Sci.* **16** 1–199
Daneshi G H and Johnson W 1977a *Int. J. Mech. Sci.* **19** 555–63
—— 1977b *Int. J. Mech. Sci.* **19** 661–72
—— 1978 *Int. J. Mech. Sci.* **20** 255–64
Hutchings I M 1976 *Int. J. Mech. Sci.* **18** 243–7
Johnson W 1972 *Impact Strength of Materials* (London: Arnold) pp 302–4
Johnson W and Reid S R 1975 *J. Mech. Engng Sci.* **17** 71–81
Jonas G H and Zukas J A 1978 *Int. J. Engng Sci.* **16** 879–903
Norris D M, McMaster W H and Wilkins M L 1976 *Lawrence Livermore Laboratory Report* UCRL-52057
Recht R F 1978 *Int. J. Engng Sci.* **16** 809–27
Richardson E G 1948 *Proc. Phys. Soc.* **61** 352–67
Soliman A S, Reid S R and Johnson W 1976 *Int. J. Mech. Sci.* **18** 279–84
Tate A 1967 *J. Mech. Phys. Solids* **15** 387–99
—— 1977 *Int. J. Mech. Sci.* **19** 121–3