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# INCREASING THE IN-BORE VELOCITY MEASUREMENTS RESOLUTION USING NON "FOURIER" TIME-FREQUENCY ANALYSIS

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To make the ballistic pressure measurements using Doppler radar techniques, which doesn't require a sensor in the armament, it's necessary to measure the in-bore velocity, and as a consequence its acceleration and pressure are obtained.
The in-bore velocity varies from zero to hundreds of m/s in a time lapse in the order of miliseconds, which corresponds to a variation in frequency from zero to some hundreds of kHz in the Doppler signal analysis. It is well known from signal processing theory, that STFT technique, which is widely used in this analysis, does not produce a good resolution for chirp signals of this type.
Alternative time-frequency techniques are applied to the in-bore Doppler shift signal, in order to improve the measurement resolution. The results for the different techniques were compared.

## INTRODUCTION

The present work deals with the ballistic pressure measurements using Doppler radar techniques, which does not require the use of a sensor into the armament. The system sensor used to measure the in-bore velocity consists of an antenna transmitting/receiving microwave radiation focused onto a mirror placed on the bore axis. The radiation is reflected back along the same path to the antenna transmitting/receiving unit. When the projectile starts moving, the reflected radiation will contain a Doppler shift proportional to the projectile velocity. The measurement continues until the projectile hits the mirror.

The data used in this work was collected at the Marambaia Brazilian Army Testing Grounds, using a DR5000 Velocity Analyzer, made by Terma Elektronik AS [1]. This equipment analyzes the reflected signal by means of Fourier Transform Technique, i.e., by the Short Time Fourier Transform (STFT), [2], which directly gives the Doppler frequency shift and, consequently, the projectile velocity as function of time.

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In this paper alternative time-frequency techniques are discussed and then applied to an in-bore real Doppler shift signal, in order to improve the measurement resolution. The results for the different techniques were compared.

## TIME-FREQUENCY ANALYSIS TECHNIQUES EMPLOYED

The techniques tested in the present work were: STFT Spectrogram, Pseudo Wigner-Ville Distribution, Choi-Williams Distribution, Cone-Shaped Distribution, Gabor Spectrogram, [3][4][5]. The purpose of these energy distributions is to divide the energy of the signal over the two description variables: time and frequency.

### STFT and the Gabor Expansion

The Gabor expansion represents a signal s[i] as the weighted sum of the frequency-modulated and time-shifted function h[i]:

$$s[i] = \sum_{m} \sum_{n=0}^{N-1} C_{m,n} h[i - m\Delta M] e^{j2\pi n1/N} \tag{1}$$

where the Gabor coefficients $C_{m,n}$ are computed by the STFT:

$$C_{m,n} = STFT(m\Delta M, n) = \sum_{i=0} s[i]\gamma^{*}[i - m\Delta M] e^{-j2\pi ni/N} \tag{2}$$

where N denotes the number of frequency bins, and $\Delta M$ denotes the time sampling interval. As long as its dual function h[i] exists, it's possible to use any funtion as $\gamma[i]$.

### STFT Spectrogram

The STFT spectrogram is defined as the modulus of the STFT:

$$SP[m\Delta M, n] = \left| \sum_{i=0} s[i]\gamma[i - m\Delta M] e^{-j2\pi ni/N} \right|^2 \tag{3}$$

where $N$ represents the number of bins, and $\Delta M$ represents the time sampling interval. The STFT based spectrogram is simple and fast, but suffers from the window effect, which is well illustrated by [4], where they used a three-tone test signal with different time locations to show that with a narrowband window, the time-dependent spectrum has a high frequency resolution but a poor time resolution. While with a wideband window, the time-dependent spectrum has a poor frequency resolution but a high resolution in time. This is the procedure employed at Marambaia's Testing Grounds, using the DR5000 velocity analyzer. The results are not very good due to the lack of a good frequency resolution.

## Wigner-Ville Distribution and Pseudo Wigner-Ville Distribution

For a signal s[i], the Wigner-Ville distribution (WVD) is:

$$WVD[i,k] = \sum_{m=-L/2}^{L/2} R[i,m]e^{-j2\pi km/L} \tag{4}$$

where the function *R[i,m]* is the instantaneous correlation given by:

$$R[i,m] = z[i+m]z^*[i-m], \tag{5}$$

The WVD can also be computed by:

$$WVD[i,k] = \sum_{m=-L/2}^{L/2} \Re[i,m]e^{j2\pi km/L}, \tag{6}$$

where:

$$\Re[i,m] = Z[i,m]Z^*[i-M] \tag{7}$$

and $Z$[k] denotes the Fourier transform of z[i]. The Wigner-Wille distribution is simple, fast and has a very good resolution. However, if the analyzed signal contains more than one component, the WVD method suffers from crossterm interference. Therefore, it could be alleviated, assigning different weights to the instantaneous correlation $R$[i,m] to supress the less important parts and enhance the fundamental ones.

There are two methods to do it. The first is in the time domain:

$$PWVD[i,k] = \sum_{m=-L/2}^{L/2} w[m]R[i,m]e^{-j2\pi km/L} \tag{8}$$

which is known as the Pseudo Wigner-Ville distribution (PWVD). Usually, the window w[m] is gaussian.

The second method is in the frequency domain:

$$WVD[i,k] = \sum_{m=L/2}^{L/2} H[m]\Re[i,m]e^{j2\pi km/L} \tag{9}$$

The eq. (9) is equivalent to:

$$PWVD[i,k] = \sum_{m=-L/2}^{L/2} \left( \sum_{n} h[n]R[i-n,m] \right) e^{-j2\pi km/L} \tag{10}$$

where $h[n]$ is the Fourier inverse transform of $H[m]$ in eq. (9).

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## Cohen Class

Leon Cohen [6] developed the distribution represented by the eq. (11), which took his name:

$$C[i,k] = \sum_{m=-L/2}^{L/2} \sum_{n} \Phi[n,m] R[i-n,m] e^{-j2\pi km/L} \tag{11}$$

the term $\Phi[i,m]$ denotes the kernel function. Comparing eq. (8) and eq. (10) with eq. (11), the windows $w[m]$ and $h[m]$ are particular cases of the $\Phi[i,m]$, as it occurs in most of the quadratic representations.

## Choi-Williams Distribution

When the kernel is defined as in the eq. (12), the distribution is called Choi-Williams (CWD)[3].

$$\Phi[i,m] = \sqrt{\frac{\alpha}{4\pi m^2}} e^{-\alpha i^2/(4m^2)} \tag{12}$$

By adjusting the parameter α in eq. (12), it's possible to balance the crossterm interference and time-frequency resolution. The greater α, less smoothing. The CWD suppress a lot of the crossterm interference between autoterms with different time and frequency centers. However, this technique can't reduce the components with same frequency or localized at the same time. The CWD is a very slow algorithm.

## "Cone-Shaped" Distribution

When the kernel is defined as the eq. (13), it results in a distribution called Cone-Shaped, which, as the CWD, could reduce the crossterm interference. The algorithm is faster than the CWD [3].

$$\Phi[i,m] = \begin{cases} e^{-\frac{\alpha m^2}{c}} & \text{for} \quad i < |m|, \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

## Gabor Spectrogram

Complementing the Pseudo Wigner-Ville distribution, it's possible to preserve the terms with major energy contributions and remove the others with minor contributions. This representation is called as Gabor Spectrogram and it's represented by eq. (14). Where $WVD_{h,h'}[i,k]$ represents the WVD of frequency modulated gaussian functions and D is the degree of smoothing.

$$GS_D[i,k] = \sum_{|m-m'|+|n-n'|\le D} C_{m,n} C_{m',n'} WVD_{h,h'}[i,k] \tag{14}$$

For $D=0$, $GS_0[i,k]$ is non-negative and is similar to the STFT spectrogram. As D goes to infinity, the Gabor spectrogram converges to the WVD.

A comparative Table between these algorithms [4] is showed on Table 1.

Table 1

| Method | Velocity | Resolution and Crossterm Description |
|---|---|---|
| STFT Spectrogram | Fast | Poor Resolution, Robust and Non-negative |
| Pseudo Wigner-Ville Distribution (PWVD) | Fast | Extremely high resolution for a few types of signals. Severe Crossterms |
| Choi-Williams Distribution (CWD) | Very slow | Less crossterms than PWVD |
| Cone-Shaped Distribution | Slow | Less crossterms interference than PWVD and CWD |
| Gabor Spectrogram | Moderate | Good resolution, Robust and Minor crossterms |

## EXPERIMENTAL RESULTS

Using the experimental Doppler signals that were collected, a qualitative comparison was made using the time-frequency techniques described above. Then, from the Doppler signal the corresponding time-velocity graph was obtained and compared with each other.

Fig. 1 shows an example of the analysis when the STFT spectrogram was used and confirmed a not very good frequency resolution as expected. The window used in this case had 256 points and, as described before, the frequency resolution is not increased simply by adding points to the window.

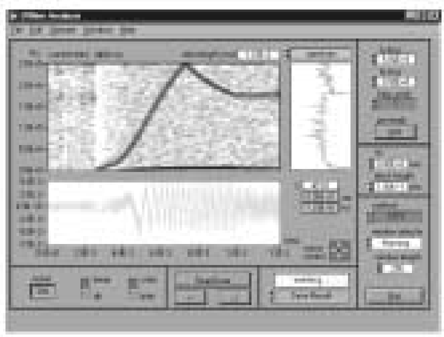

Figure 1: STFT Analysis of the Doppler signal.

The same signal was analysed by the other techniques described above, and it was confirmed the resolution enhancement and the flexibility added to the analysis. Fig. 2 shows the results using the PWVD and confirms the worst results due to the crossterms interference. Fig. 3, Fig. 4 and Fig. 5 shows the very good results obtained using the Cone-Shaped, Choi-Williams and Gabor respectively. Comparing with Fig. 1, a thinner velocity curve were computed indicating a better resolution and definition. The best results were obtained using the Gabor and Cone-Shaped techniques, where the windows has a length of 256 samples.

All these results were obtained using the software Labview and its Joint Time-Frequency Analysis Toolkit, both from National Instruments, USA [4].

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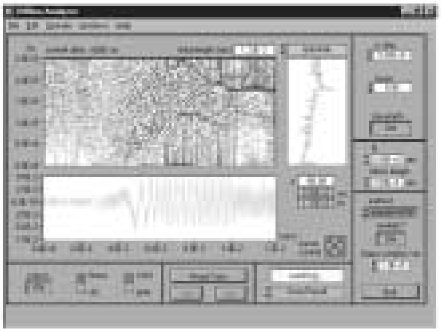

Figure 2: PWVD analysis of the Doppler signal.

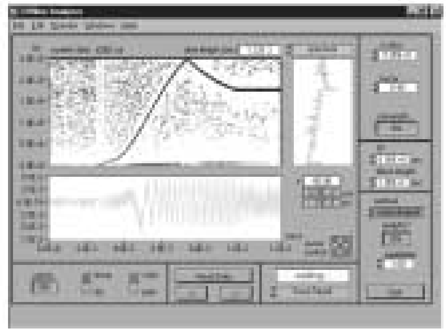

Figure 3: Cone-Shaped analysis of the Doppler signal.

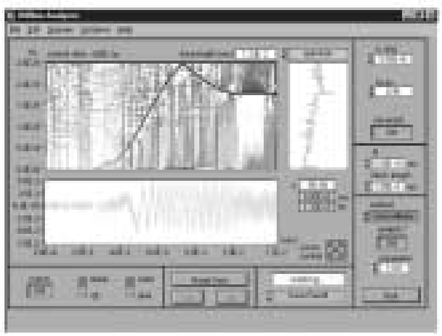

Figure 4: Choi-Williams analysis of the Doppler signal.

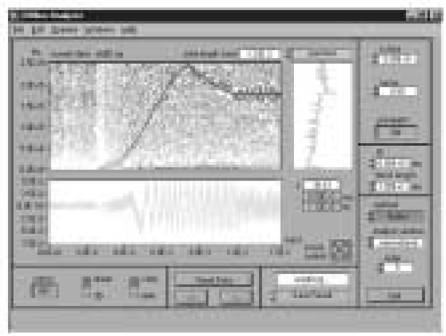

Figure 5: Gabor analysis of the Doppler signal.

## CONCLUSIONS

Some algorithms were showed as an alternative to the traditional Fourier method utilized by the equipments used by the Brazilian Testing Ground. In fact, there are other time-frequency algorithms, which could be used.

When analyzing in-bore Doppler radar signals it's interesting to have an option to choose the best algorithm to take the best time-velocity diagram as showed in this work.

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