19th International Symposium of Ballistics, 7–11 May 2001, Interlaken, Switzerland

CAVITY EXPANSION THEORY APPLIED TO PENETRATION OF TARGETS WITH PRE-DRILLED CAVITIES

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An analytical model for penetration of rigid projectiles into concrete targets containing pre-drilled cavities is developed. The model is based on a modification of cavity expansion theory. It is compared with other semi-empirical models and two sets of experimental data. However, due to lack of triaxial concrete data from the experiments, the concrete properties had to be estimated from an empirical relation. Despite this, the model is seen to give qualitative agreement with experiments, but further research and experiments are needed to examine the quantitative predictions of the analytical model.

INTRODUCTION

An important problem in penetration mechanics is to determine the effect of having a weakened target, maybe because the first stage of a tandem charge has damaged the target before impact of the main projectile.

As a first approach to the general problem, we consider a situation of a projectile penetrating a target containing a pre-drilled cylindrical cavity. Previous work on this topic has mainly been based on empirical and numerical studies. In this paper we attempt to model the problem analytically, using the penetration theory based on cavity expansion.

OVERVIEW OF THE PROBLEM

Our situation is illustrated in Figure 1. A projectile with radius a is impacting a target with a pre-drilled cavity of radius *b*. It will be convenient to define the relative cavity radius (or diameter) by $R=b/a$. Formulas for penetration of targets with pre-drilled cavities can be given either in terms of absolute penetration depth x, or as relative penetration depth $X = \frac{x(R)}{x(R=0)}$. Both points of view are of interest and will be presented in this paper.

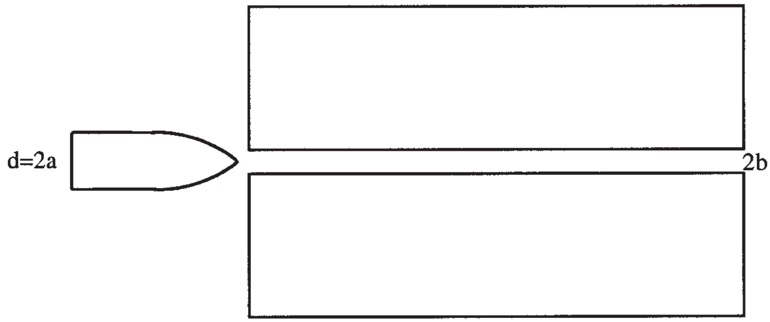

Figure 1: Penetration of a projectile of radius a into a target with pre-drilled cavity of radius b.

PREVIOUS WORK

The problem of having a concrete target with pre-drilled cavity was first examined by Murphy [1]. His approach was based on Bernard's empirical equation [2] for penetration into rock, which was modified in the following way:

$$\left(\frac{x}{d}\right)_{Murphy} = \frac{1}{1-R^2}\left(0.0254\frac{mv_0}{d^3\sqrt{\rho\sigma_c}}\right) \tag{1}$$

where $d=2a$ is the projectile diameter, m is the mass, v_0 the impact velocity, ρ is the concrete density and σ_c is the compressive strength. However, the relative penetration depth X is easily seen to take on a much simpler form:

$$X_{Murphy} = \frac{1}{1-R^2} \tag{2}$$

Folsom [3] modified the ACE empirical equation [4] with two unknown constants that were empirically determined according to his experiments. His final result was:

$$\left(\frac{x}{d}\right)_{Folsom} = \frac{1-0.38R^2}{1-R^2}\left(5.47\cdot 10^{-4}\frac{mv^{1.5}}{\sigma_c d^{2.785}}\right) - \frac{4}{1-R^2}f(\psi,R) + \sqrt{\psi-\frac{1}{4}}$$
$$f(\psi,R) = -\frac{g^3}{3} + \left(\frac{1-R^2}{4} - \psi + 2\psi^2\right)g + \frac{(1-2\psi)}{2}\left(g\sqrt{\psi^2-g^2} + \psi^2\arcsin\left(\frac{g}{\psi}\right)\right) \tag{3}$$
$$g(\psi,R) = \sqrt{\psi(1-R) - \frac{(1-R)^2}{4}}$$

For sufficiently large velocities, the first term dominates over the two other terms, and the relative penetration depth is easily seen to reduce to a quite simple expression:

$$X_{Folsom} = \frac{1-0.38R^2}{1-R^2} \tag{4}$$

The same problem was later examined by Mostert [5], who used a combination of numerical and experimental observations to independently rederive Equation (2).

MODIFIED CAVITY EXPANSION THEORY

In this paper we slightly modify the penetration theory of cavity expansion to make it applicable to penetration of targets containing pre-drilled cavities. A similar approach has recently been suggested independently by Szendrei [6]. See Teland [7] for a review of cavity expansion theory.

When an initial cavity of radius *b* is present in the target, the force on the projectile can be found by integrating only over the part of the surface that is in contact with the target material.

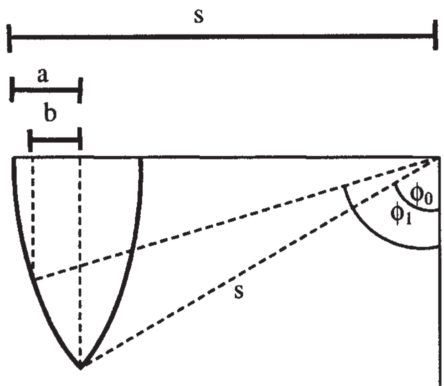

Figure 2: The projectile and initial cavity geometry.

This amounts to integrating from ϕ_1 to $\pi / 2$, instead of from ϕ_0 to $\pi / 2$ as in regular cavity expansion theory, where the angles ϕ_1 and ϕ_0 are defined in Figure 2. The expression for the force can now be written as:

$$F = -2\pi s^2 \int_{\phi_1}^{\pi/2} p_r(v,\phi)\left(\sin\phi - \sin\phi_0\right)\cos\phi \, d\phi \tag{5}$$

where the function p_r is an estimate of the radial stress on the projectile during penetration. It is found from cavity expansion theory and can in many cases be approximated by:

$$p_r = A + Bv^2 \cos^2\phi \tag{6}$$

where A and B are constants depending on properties of the target material, v is the projectile velocity and the angle ϕ defines the position on the projectile surface. After some cumbersome algebra, our final result for the force becomes:

$$F = -\left(\alpha + \beta v^2\right)$$

$$\alpha = \pi a^2\left(1 - R^2\right) \qquad \beta = \frac{\pi a^2 B}{24\psi^2}\left(\left(8\psi - 1\right) - R^2\left(6\left(4\psi - 1\right) + 8R\left(1 - 2\psi\right) - 3R^2\right)\right) \tag{7}$$

Equation (7) is consistent with regular cavity expansion theory when $R = 0$. For a given material model, A and B can be calculated analytically or numerically directly from cavity expansion theory. For concrete, they will typically depend on the elastic parameters, triaxial yield curve and other properties of the corresponding target material. Sometimes a complete concrete description is unavailable, in which case A and B can be approximated by an empirical expression derived by Forrestal et.al. [8–9]. This is seen to be only dependent on σ_c:

$$A = S\sigma_c \quad S = 82.6\left(\frac{\sigma_c}{10^6}\right)^{-0.544} \quad B = \rho \tag{8}$$

PENETRATION DEPTH

The penetration process can be divided into two phases. In the initial penetration phase, the projectile has either not interacted with the target yet (because of the cavity) or only a part of the nose is in contact with the material. The phase ends when the projectile has penetrated a distance x_{init} and is completely surrounded by target material. We will find a numerical solution (see Berthelsen [10]) for this phase as it turns out to be impossible to describe analytically.

After the initial penetration phase, Equation (7) for the total force on the projectile can be combined with Newton's 2nd law to calculate the projectile deceleration. The penetration depth in this phase is then eventually found to be given by:

$$x_1 = \frac{m}{2\beta}\ln\left(1+\frac{\beta}{\alpha}v_1^2\right) \tag{9}$$

where v_1 is the velocity of the projectile after the initial penetration phase. In cases where the initial phase can be neglected, we can put $v_1 = v_0$. Assuming this, we have the following expression for the normalised penetration depth X:

$$X = \frac{x_1(R) + x_{init}(R)}{x_1(R=0)} \tag{10}$$

For low velocities, and ignoring the contribution from x_{init}, Equation (10) can be shown to approach the result of Equation (2).

COMPARISON WITH EXPERIMENTAL DATA

Both Folsom and Mostert have performed penetration experiments into concrete targets with pre-drilled cavities of various diameters. In this section we will compare our analytical theory with their experimental data.

Experimental data from Folsom

Folsom performed experiments with 5.93 kg projectiles of diameter 88.6 mm and nose curvature $\psi = 1.25$. The concrete had a compressive strength of 48.5 MPa, a density of 2370 kg/m^3 and the impact velocity was approximately 206 m/s. The diameter of the concrete targets was 40.64 cm, which gives a ratio between target and projectile diameter of only 4.59. This was probably not enough to stop boundary effects from increasing the penetration depth, as discussed in Teland and Sjøl [11].

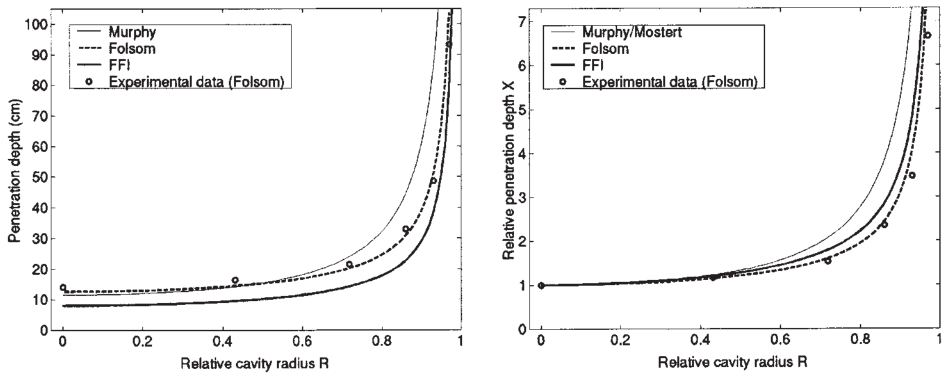

Figure 3: Penetration depth as a function of cavity radius for an 88.6 mm projectile impacting 48.5 MPa concrete targets.

ln Figure 3 we have plotted the absolute and relative penetration depth as a function of the relative cavity radius R. Murphy's formula is seen to overestimate the penetration depth for large R in both cases, whereas Folsom's formula seems to be pretty accurate, especially for large initial cavities. This is not surprising as Folsom's formula was created on the basis of curve fitting to exactly these experimental data. The cavity expansion approach is seen to consistently underestimate the absolute penetration depth, which is however to be expected if boundary effects were present. It overpredicts the relative penetration depth, which is related to the underprediction of $x(R = 0)$.

Experimental data from Mostert

Mostert has performed several experiments with projectiles impacting reinforced concrete targets containing initial cavities of various diameters. According to Mostert [12], the projectile had a mass of 141.6 g, diameter 20 mm and $\psi = 2.11$. The concrete had a compressive strength of 20 MPa and an estimated density of 2000 kg/m^3.

The targets were 30 cm thick and had a diameter of 30 cm, which gives a target/projectile diameter ratio of 15. Boundary effects should therefore not be present in the experiments, except perhaps in the cases of large initial cavity when the projectile almost perforated the target.

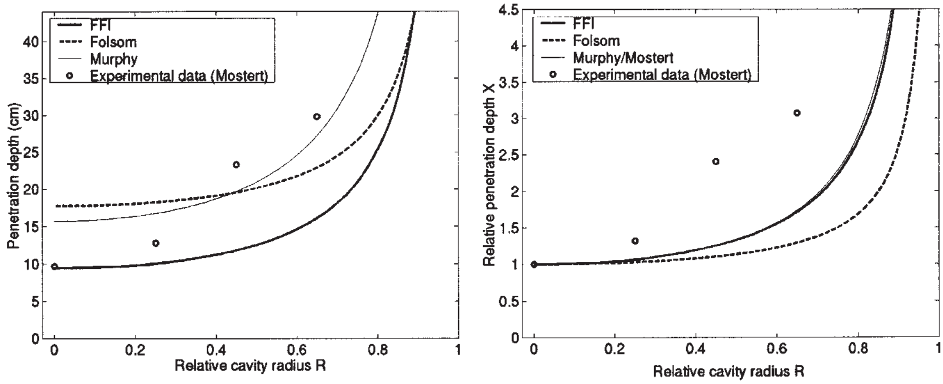

Figure 4: Penetration depth as a function of cavity radius for a 20 mm projectile impacting 20 MPa concrete targets.

Mostert fired two shots for each initial cavity diameter, but the same velocity of 350 m/s was not always obtained. In our comparison we have used the data paints which were closest to 350 m/s.

In Figure 4 we have plotted the penetration depth as a function of cavity radius. It is seen that none of the formulas agree very well with all the experimental data. The cavity expansion approach, however, is seen to give good result for *R=0*, but underpredicts penetration in the other cases. This could be due our applied concrete model being inaccurate and possible boundary effects at the rear of the target. Folsom's equation is seen to very much underestimate the relative penetration depth.

Mostert also performed experiments with the same targets but only 15 cm deep initial cavities. The cavity expansion based theory can easily be adapted to this case as well. If the projectile penetrates deeper than 15 cm, we only have to switch to normal theory (*R=0*).

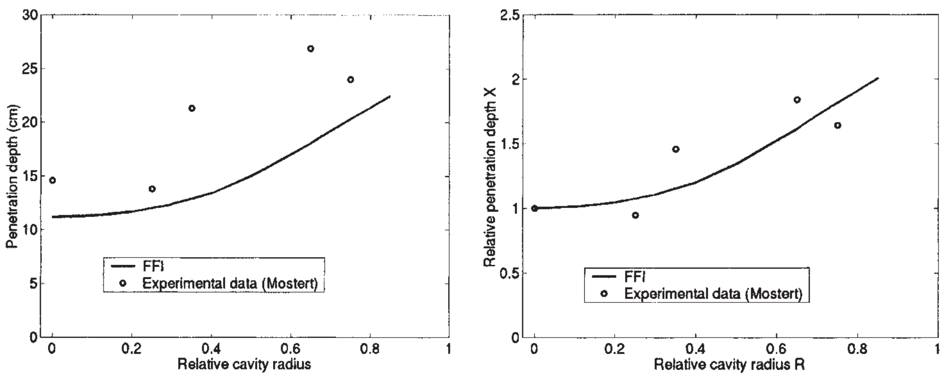

Figure 5: Penetration depth as a function of cavity radius for a 20 mm projectile impacting 20 MPa concrete targets with initial cavities of 15 cm.

In Figure 5 we have plotted the absolute and relative penetration depth as a function of cavity radius compared to Mostert's experimental data for 15 cm deep cavities. The other formulas were not applicable to this case. It is seen that the absolute penetration depth is somewhat underestimated by the formula, while the relative penetration depth seems to fit the data quite well. Again this could be explained by the applied concrete model being inaccurate.

SUMMARY

We have presented an analytical method for calculating penetration into a target containing a pre-drilled cavity. This should be considered as a first approximation to the full problem of penetration of a tandem charge.

The model has been compared with two sets of experimental data and the results so far indicate that it might be able to predict the penetration depth when an initial cavity is present. However, the accuracy of the model is uncertain since the complete triaxial properties of the concrete used in the experiments were not known, and the material constants of the model therefore had to be estimated through an empirical relation. Also, boundary effects might have been present in some of the experiments, which again makes it difficult to compare the experimental results with the predictions of the model. It is clear that further research and experiments are needed on this topic.

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