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# COMPARING ALTERNATE APPROACHES IN THE SCALING OF NATURALLY FRAGMENTING MUNITIONS

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In a seminal study, Mott developed relations for calculating the fragmentation characteristics of naturally fragmenting cylinders. Here, the issue of scaling is further explored through application of the later theoretical efforts of Mott, along with other more recent theoretical studies. Scaling predictions are compared with recent fragmenting cylinder data.

Mott [1, 2] undertook one of the earliest, and probably most familiar, theoretical investigations of the dynamic fragmentation of exploding cylindrical shells. Out of this study the two analytic expressions, in Mott's original symbols, have emerged as commonly accepted scaling relations for fragmenting shells (e.g., Cooper [3]).

$$n(M)\,dM = Be^{-M/M_A}\,dM\,, \tag{1}$$

$$M_A = Ct^{5/6}d_2^{1/3}\left(1+t/d_2\right). \tag{2}$$

Equation (1) is a description of the statistical fragment size distribution while Equation (2) provides the fragment size scale parameter for the preceding distribution. In these equations $B$ and $C$ are constants while $n(M)$ is fragment number and $M = m^{1/2}$ where $m$ is fragment mass. Cylinder shell thickness and inner shell radius are $t$ and $d_2$, respectively. It is seldom noted that Mott himself, both directly and tacitly, refuted the above two relations in later results of the same study. Both theoretical and experimental alternatives were offered in his later efforts which are rich in the depths to which issues governing dynamic fragmentation were pursued.

Here we propose an alternative set of relations for the fragmentation of explosively expanding shells,

$$N(m) = N_o e^{-(m/\mu)^{\beta}}\,, \tag{3}$$

$$\mu = A\rho T^3\left(\frac{R}{T}\right)^{2s/\alpha}\left(\frac{G/T^{2-\alpha}}{\rho V^2}\right)^{s/\alpha}. \tag{4}$$

In the preceding Equation (3) $N(m)$ and $N_o$ are the cumulative and total fragment number, respectively, while $\mu$ (dimensions of mass) is the fragment size distribution scale parameter and $\beta$ (the distribution shape parameter) is a constant with a value within the approximate range $1/3 \leq \beta \leq 3$. In Equation (4) $A$ is a fragment aspect ratio, $s$ and $\alpha$ are constants with $s$ equal to either 2 or 3 depending on whether the dominant fragment size exceeds, or does not exceed, the case thickness. A theoretical range for $\alpha$ is $2 \leq \alpha \leq 3$. $R$ is the nominal radius of the expanding shell and $T$ is shell thickness; $G$ and $V$ are fracture resistance and radial expansion velocity, respectively.

It is critical to here emphasize that Equations (3) and (4) are not counter to the theoretical analysis of Mott. They are, in fact, one of several representations that emerge from the later ideas presented in Mott's classic papers on fragmentation. More recent fragment size theories can also be incorporated into Equations (3) and (4) [4, 5]. It is the intent of this short paper to emphasize the theoretical rather than applications-oriented issues governing dynamic fragmentation with particular focus on Mott's theoretical work.

In pursuing a statistical description of shell fragments, Mott was influenced by munitions fragment size data available to him, in addition to the then recent theoretical work of Lineau [6] in which random fragmentation of a line led to the distribution in lengths $l$ of the form,

$$p(l)dl = \frac{1}{\lambda} e^{-l/\lambda} dl \, . \tag{5}$$

Mott reasoned from Lineau's theory that in the fragmentation of a thin shell that the size scale $l \sim m^{1/2}$ might continue to be the random variable. Mott's fragment distribution law in Equation (1) follows directly from this argument. Here we pursue a more general development based on principles of survival, or hazard, statistics [7]. Accordingly, a body of mass $M$ broken into $N$ fragments of random mass $m$ has the distribution,

$$p(m)dm = h(m) e^{-\int_o^m h(m)dm} dm \, , \tag{6}$$

where $h(m)dm$ is the random chance of fragment rupture into the mass interval $m$ to $m+dm$. The statistical fragment size distribution is constrained by specifying the functional dependence of $h(m)$ on the fragment mass $m$. This is done by either exploring additional physics specific to the fragmentation process, or by hypothesis to be tested by data such as the earliest assumption pursued by Mott [1, 2] mentioned above.

For example, given no additional insight into the fragmentation process there is little reason to assume a bias of $h(m)$ toward either the large or small masses and the simplest assumption is $h(m) = h_o$ a constant. Thus $p(m)$ in Equation (6) becomes,

$$p(m) = h_o e^{-h_o m} \, , \tag{7}$$

and the cumulative fragment number distribution is,

$$N(m) = N_o e^{-m/\mu} \, , \tag{8}$$

where $\mu = 1/h_o$ is the distribution scale parameter. Applicability of this distribution to dynamic fragmentation has been suggested by Grady and Kipp [8].

Alternately, following Mott's assumption of $l \sim m^{1/2}$ and $dl \sim \frac{1}{2} m^{-1/2} dm$ it follows that,

$$h(m) = \frac{1}{2\mu}\left(\frac{m}{\mu}\right)^{-1/2}, \tag{9}$$

resulting in the cumulative fragment number distribution initially favored by Mott,

$$N(m) = N_o e^{-(m/\mu)^{1/2}}. \tag{10}$$

In the present development we consider the power law functional dependence,

$$h(m) = \frac{\beta}{\mu}\left(\frac{m}{\mu}\right)^{\beta-1}, \tag{11}$$

which, when integrated, leads to the size distribution scaling relation of the Weibull form,

$$N(m) = N_o e^{-(m/\mu)^{\beta}}. \tag{12}$$

This relation clearly encompasses the previous examples. Namely, $\beta$ = 1/2 corresponds to the distribution arrived at by Mott and Linfoot [9] while $\beta$ = 1 corresponds to that suggested by Grady and Kipp [8]. Generality of the parameter $\beta$ for a munitions-specific scaling equation is warranted for a number of reasons. Mott and Linfoot [9] argued that, when the fragment distribution was dominated by fragments of a size less than the case thickness, $\beta$ = 1/3 was probably more appropriate. For a specific munitions system, a range of expansion strain rates will lead to statistical heterogeneity (a different size parameter at different positions along the munition case). This breakup feature will broaden the distribution leading to smaller effective values of $\beta$. On the other hand, a degree of case scoring, or other system processing, with the intention of biasing the distribution toward a unique size, has the effect of increasing the distribution shape parameter $\beta$. (Note that as $\beta$ approaches infinity Equation (12) approaches a Heaviside function.) Additionally, Mott's later fragmentation theory suggests higher values for $\beta$.

To determine the mass scale parameter in the second scaling relation (Equation (2)) Mott joined two elemental theories. First a characteristic fragment size was determined by equating the work required to open a single fracture with the stretching kinetic energy on both sides of the fracture point. Expansion velocity of the shell introduced through the kinetic energy expression was then solved for through an explosive energy-expansion kinetic energy balance not unlike the methods developed by Gurney [10].

The present Equation (4) is derived similarly except specific models for the fragmentation resistance, $G$ and the expansion velocity $V$ are not explicitly introduced. Effective velocities can be calculated from hydrocode simulations as well as with Gurney methods. Fragmentation resistance may be estimated through physics-based theories such as Curran et al. [11], Kipp and Grady [5] or through the later statistical theory of Mott [1, 2].

This later theoretical approach of Mott pursued in the last of the 1943 reports introduced new physics including a markedly stronger physics-based approach to the statistical processes. Recognizing that fracture in an expanding cylindrical shell occurred initially as longitudinal cracks after becoming plastic under circumferential tension, he idealized the problem to that of a ring of metal at radius *R* expanding at a velocity *V* and stretching plastically at a tensile flow stress *Y*. As plastic strain increases within the expanding body a probabilistic relation is proposed which expresses the chance of fracture at random positions on the rings. Upon fracture at a point, waves propagate away relieving the tensile stress and precluding further fracture in regions encompassed by the waves. This solution assuming ideal plasticity ahead of the wave and rigid behavior behind showed that the Mott wave reached a distance, $x = \sqrt{2Yt / \rho\dot{\varepsilon}}$ from the point of fracture at a time *t* after fracture. Later elastic-plastic wave solutions of Lee [12] verified that Mott's approximate solution was, indeed, very good.

With the solution in hand for establishing that rate at which stress is relieved in the neighborhood of fractures Mott proceeded to pursue specific analytic forms for the dynamic random fracture process. Here we have carried through Mott's analysis assuming that the chance of fracture occurrence per unit length and unit strain interval is given by the power-law function,

$$f(\varepsilon) = \frac{n}{\sigma}\left(\frac{\varepsilon}{\sigma}\right)^{n-1} . \tag{13}$$

The probabilistic fracture relation (Equation (13)) can be combined with the stress release expression to establish the number of fractures which occur in an expanding cylindrical or ring-shaped body at a characteristic expansion strain rate $\dot{\varepsilon} = V / R$. The analysis is too extensive to detail here, but essential concepts have been reported in Grady [13]. We find for the number of fractures per unit length,

$$N = \alpha_n \left(\frac{\rho\dot{\varepsilon}^2}{Y\sigma}\right)^{\frac{n}{2n+1}} . \tag{14}$$

The coefficient $\alpha_n$ is a slowly increasing function of *n* with $\alpha_1 \cong 0.6$ while $\alpha_{25} \cong 2.0$. Note the physical properties in Equation (14) governing the characteristic fragment size. The parameter $\sigma$ is a measure of the strain to failure while *n* determines the standard deviation about this strain. Similar results were obtained by Mott.

Mott then proceeded to determine the distribution in fracture spacings (fragment lengths) using the same statistical fracture theory. This procedure is best detailed in his open literature publication of his earlier work [14]. Mott chose to demonstrate the character of the resulting size distribution through a direct hand analysis. A comparable analytic solution for the distribution has been found for the special case of a uniform fracture probability function, $n = 1$ above [12]. It is also readily shown that the cumulative number distribution in fragment lengths is analytically represented by,

$$N(l) = N_o e^{-(l/\lambda)^3} , \tag{15}$$

based on a hazard statistics description of the fracture physics which contrasts markedly with the Lineau distribution in Equation (5). Mott's assumptions of $l \sim m^{1/3}$ or $l \sim m^{1/2}$ for case fragmentation would then lead to $\beta$ of 1 or 3/2 in Equation (12), demonstrating the increased value of $\beta$ resulting from Mott's later statistical fragmentation theory.

Recent explosive fragmentation experiments on heat-treated and as-received AerMet 100 steel have been performed. Details are addressed elsewhere in the present proceedings. Here we examine fragmentation and scaling within the scope of the theoretical work of Mott as well as a more recent energy-based fragmentation theory [11]. Experiments examined here were performed on right circular cylindrical shells with length equal to inner diameter. Replica scaling of the test geometry was investigated. Individual tests are here identified as full scale (FS), length ≅ 20 cm, or half scale (HS), length ≅ 10 cm. Wall thickness to inner radius ratio was $T/R = 0.08$.

Cumulative fragment number distributions for the half-scale and full-scale heat-treated tests are shown in Figure 1. Distribution parameters from a best fit of Equation (3) to the fragmentation data for both heat-treated (ht) and as-received (ar) steel are provided in Table 1. The number parameter is not independent but is determined from the total recovered mass of fragments $M$ through $N_o = M / \mu\Gamma((1+\beta)/\beta)$ where $\Gamma(\ )$ is the gamma function.

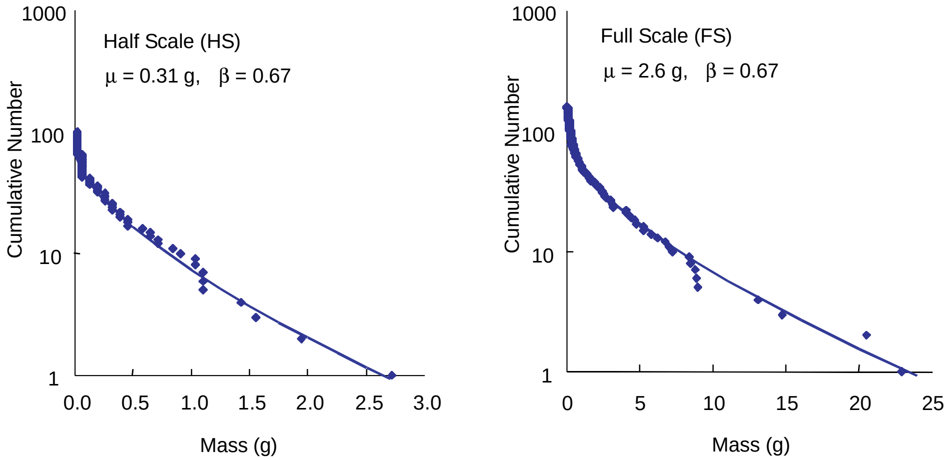


Figure 1. Fragment distributions for heat-treated AerMet® 100 fragmentation test.

Table 1. Fragment distribution parameters

| | HS (ht) | FS (ht) | HS (ar) | FS (ar) |
|---|---|---|---|---|
| Scale parameter ($\mu$) | 0.31 g | 2.6 g | 0.70 g | 5.2 g |
| Shape parameter ($\beta$) | 0.67 | 0.67 | 0.55 | 0.85 |
| Number parameter ($N_o$) | 66 | 79 | 42 | 88 |

Parameters assumed for AERMET-100 steel: $Y$ = 1.5 GPa, $K_c$ = 70 MPa/m$^{1/2}$, $\rho$ = 7900 kg/m$^3$, $c$ = 5000 m/s, $\sigma$ = 0.3, $n$ = 25.

Scale parameter $\mu$ for the four experiments normalized by $\rho T^3$ (0.52 g (HS), 4.4 g (FS)) is plotted in Figure 2. Tests on heat-treated steel are clearly consistent with replica scaling as is evident by the constant value of the normalized scale parameter. Distributions for tests on as-received steel do not as tightly constrain $\mu$ as in the heat-treated case, but values here are not inconsistent with replica scaling. Observations on scaling have implications on the several theories predicting the fragmentation resistance $G$ in Equation 4.

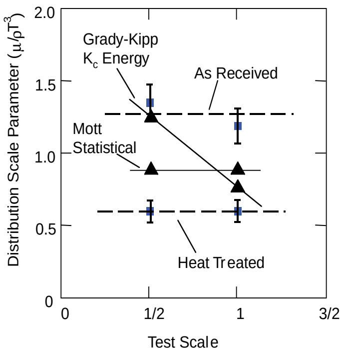


Figure 2. Distribution size scale parameter for cylinder fragmentation tests.

The energy-based theory [5] based on a fracture toughness $K_C$ fragmentation resistance predicts,

$$G = 24K_c^2 / \rho c^2 , \tag{16}$$

with exponent $\alpha$ = 3 in Equation (4). Mott's statistics-based theory, on the other hand, yields, with exponent $\alpha = 2$,

$$G = Y\sigma / \alpha_n^{(2n+1)/n} . \tag{17}$$

Calculations of the scale parameter $\mu$ from Equation (4) based on either the $K_C$ energy expression (Equation (16) or the Mott statistical expression (Equation (17)) are compared with experiment in Figure 2.

While AerMet 100 material properties are not fully quantified, precise values are not critical to the present observations, and the very reasonable values used are included in Table 1. No attempt has been made to account for the obvious difference in strength properties of heat-treated and as-received steels. An expansion velocity $V$ = 2000 m/s and an aspect ratio of $A$ = 1.5 assumed in Equation (4) are consistent with measured velocities and inspection of recovered fragments.

Looking at the results shown in Figure 2, we can see that fracture resistances determined from Mott's statistical theory and the energy-based theory of Kipp and Grady provide equally reasonable quantitative predictions of the experimental fragment size scale $\mu$. The Mott theory is consistent with the observed replica scaling, however, whereas the fracture toughness based theory is not. It is well known (e.g., Lawn and Wilshaw [15]) that for ductile fracture in metals, yield stress and fracture toughness can be related through a process zone length scale. When this length scale approaches characteristic specimen dimensions (shell case thickness for example) size effects can be observed. Present replica scaling results for fragment size suggest that the effective toughness must depend on case thickness. Considering realities of the complex fracture mechanisms in expanding cylinder fragmentation including cooperative interaction of adiabatic shear banding along with shear and tensile fracture such dependence should probably not be surprising.

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