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A NOVEL APPROACH TO THE MULTIDIMENSIONAL NATURE OF VELOCITIES OF FRAGMENTS ORIGINATING FROM CONVEX SHAPED WARHEADS

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Experimental data showed that fragment velocities originating from the convex section of a warhead which was initiated at the opposite end are higher than predicted by the Gurney equation for cylindrical warheads, adapted for end effects. A novel approach for addressing the multidimensional nature of the velocities of fragments originating from the convex section of a warhead is proposed. Two limiting conditions are defined, i.e. for fragments on the circumference of a cylindrical charge and fragments on a cylindrical charge in an open-face sandwich configuration, respectively. An asymptote matching technique is then used to obtain an expression that describes the transition between the two respective asymptotes.

INTRODUCTION

Many calculations of velocities of explosively accelerated items are successfully done via sophisticated numerical procedures incorporating the equations of states of the material and strength models. However, it is often a tedious (and difficult) task to set-up the problem and even with the new generation software, solving these problems can be time-consuming. Therefore, there is still the requirement of simple empirical and/or analytical procedures to estimate the initial velocities and ejection angles of explosively accelerated items. The well known Gurney formulas [1] which are even today used in many applications, give sufficiently accurate velocity predictions in certain configurations.

The problem of handling the end-effects in cylindrical charges for fragment sleeves around the circumference of cylindrical sections has been addressed by a number of authors. Most notably the definition of the 'relaxation coefficient', F_x, by [2] which broadened a concept used by [3] facilitated a direct reduction of the charge-to-metal mass ratio, C/M, in the standard Gurney formula:

$$v_c = \sqrt{2E}\left(\frac{M}{CF_x} + \frac{1}{2}\right)^{-1/2} . \qquad (1)$$

The F_x factor in eq. (1) was derived from numerical results of various configurations, and is dependent on *M/C* as well as *L/D*. This prediction methodology was further extended by [4] and [5].

The following expression for the relaxation factor F_x was proposed [2]:

$$F_x = \left[1 - \left(\frac{r_1}{R_1}\right)^2\right]\left[1 - \left(\frac{r_2}{R_2}\right)^2\right], \tag{2}$$

where

$$\frac{r_1}{R_1} = 1 - n_1\left(\frac{x}{D}\right)^{n_1}; \qquad \frac{r_2}{R_2} = 1 - 2n_2\left(\frac{L}{D} - \frac{x}{D}\right)^{n_2}. \tag{3}$$

The indices 1 and 2 in above equations represent the initiation end and the end opposite initiation, respectively. The numerical values of n_1 and n_2 depend on the charge-to-metal mass ratio (*C/M*) of the warhead.

With a convex shaped warhead it was found that the actual fragment velocities are higher than the predicted velocities in the curved (or convex) section of the warhead. The reason for this phenomenon is that the Gurney equation used in the calculations was derived for a long cylindrical warhead and the *C/M* used is the ratio of the whole charge mass to the total mass of the surrounding material. In the prediction model the warhead is typically divided into discs, each with a length equal to the fragment length, of which the radial *C/M* is then calculated and used in the Gurney equation to obtain the velocity of a fragment in that specific ring. This use of the Gurney equation is, however, a deviation from its 'intended' use and may result in incorrect velocities.

In eq. (1) the *C/M* was scaled to account for the reduction in velocities due to end effects. For convex shaped warheads one can argue that this correction is too severe and therefore the F_x-correction factor needs to be adjusted.

'VOLUMETRIC' CORRECTION FACTOR

The velocities in the convex part of the warhead can be corrected by artificially adjusting the *C/M* using a 'volumetric' correction factor by introducing an additional coefficient to the correction factor F_x:

$$F_x = \left[1 - \left(\frac{r_1}{R_1}\right)^2 \left(a\,\frac{R_1}{r_x}\right)^n\right]\left[1 - \left(\frac{r_2}{R_2}\right)^2 \left(b\,\frac{R_2}{r_x}\right)^n\right], \tag{4}$$

where R_1 and R_2 are the radii of initiation and opposite ends, respectively, of main charge, r_x is the radius of the main charge at distance x from the initiation end, $a;b$ and n are empirical constants. This effectively increases the magnitude of F_x and consequently the fragment velocities.

A similar model was described in [6] where the relaxation coefficient is based on the concept of [3]. In the model the following values for the empirical constants in eq. (4) were used: $n = 2$, $a = 0.7$ and $b = 0.8$. The main criticism to this model is that eq. (4) does not revert to eq. (2) if the warhead is cylindrical.

MULTI-DIMENSIONAL MODEL

The problem under consideration is multi-dimensional due to the effects of the release waves as well as to the convex shape of the warhead. When considering the ejection velocities of fragments from a convex shaped warhead one can argue there are two velocity components involved, *i.e.* the radial component and an axial component. In the calculation of the fragment velocities, using eq. (1) only the *C/M* calculated in the radial direction is used. For the convex section of the warhead it is evident that the section of the charge behind the fragments in the axial direction also contribute to the acceleration of the fragments. Consequently the *C/M* in the axial direction also needs to be incorporated into eq. (1).

Referring to Fig. 1, the *radial* charge-to-metal mass ratio, $(C/M)_R$, can be calculated as follows:

$$(C/M)_R = \frac{\rho_c r_x^2}{\rho_f\left(\left(r_x + t_{eff_R}\right)^2 - r_x^2\right)}, \tag{5}$$

where r_x is the radius of the charge at position x, ρ_c is the density of the charge and ρ_f the mean density of the surrounding material. The effective thickness of the fragment in the radial direction, t_{eff_R}, is given by

$$t_{eff_R} = \frac{t_f}{\cos\theta}, \tag{6}$$

where t_f is the thickness of the fragment and θ is the angle between the fragment and the horizontal axis. Similarly, the *axial* charge-to-metal mass ratio, $(C/M)_A$, is:

$$(C/M)_A = \frac{\rho_c x}{\rho_f t_{eff_A}}, \tag{7}$$

where x is the axial position of the fragment and the effective thickness of the fragment in the axial direction, t_{eff_A}, is given by

$$t_{eff_A} = \frac{t_f}{\sin\theta}. \tag{8}$$

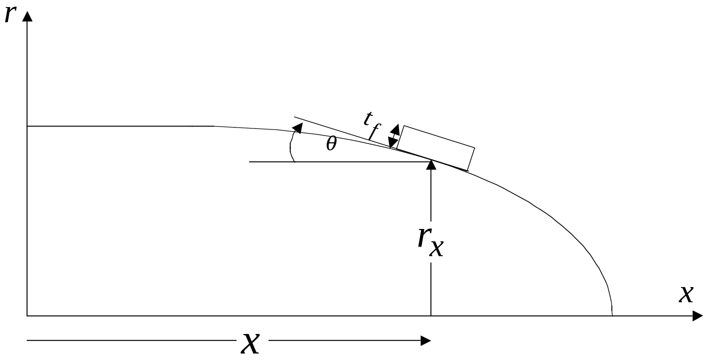

Figure 1: Schematic representation of warhead.

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From eq. (5) to eq. (8) it is clear that the *C/M* is a function of the tangent to the fragment. The axial charge-to-metal mass ratio, $(C/M)_A$, is zero in the cylindrical part of the warhead and gradually increases towards the convex section of the warhead, whereas $(C/M)_R$ is a maximum in the cylindrical part of the warhead and decreases towards the convex section. Two methods of combining the above limiting conditions will be investigated. Firstly, a single expression for the *C/M* can be obtained by combining eq. (5) and eq. (7) into a single expression using an asymptote matching technique. This expression will then be used in eq. (1) for the prediction of fragment velocities. The second approach is to "define" another expression for predicting the axial component of the velocities using $(C/M)_A$, and combine it with eq. (1) (and $(C/M)_R$) by means of an asymptote matching technique.

ASYMPTOTE MATCHING TECHNIQUE

Often in transport processes the limiting solutions for large and small values of an independent variable are known but solutions for the intermediate cases are not in closed form. Churchill and Usagi [7] proposed a general expression which interpolates between the two limiting solutions, thereby obtaining solutions for the whole range of the independent variable.

Method 1: Combined *C/M* representation

Using the asymptote matching technique described above an expression for *C/M* can be obtained which incorporates the radial as well as the axial *C/M*'s, yielding

$$C/M = \left[(C/M)_A^n + (C/M)_R^n \right]^{1/n}, \tag{9}$$

where the shifting parameter, n, determines the extent of overlap between the two physical conditions described by the two limiting expressions.

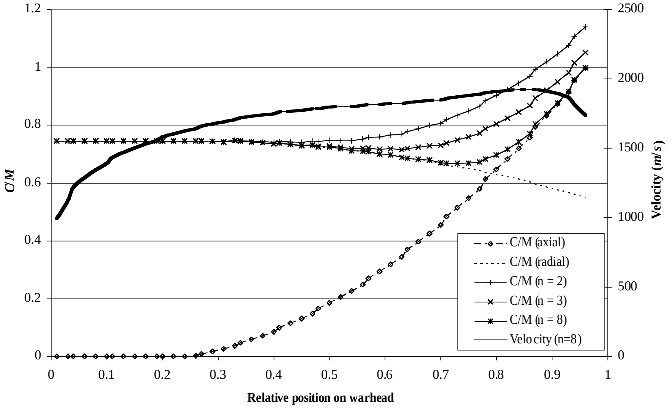

Figure 2: Method 1: Combined *C/M* representation.

In Fig. 2 the *C/M*'s, using eq. (9) for various values of *n*, are shown. As illustrated in Fig. 2, an increase in n would cause eq. (9) to follow the asymptotes more severely.

Method 2: Combined open-face sandwich and cylindrical configurations

As previously discussed, the radial velocity component may be addressed by the Gurney equation for cylindrical warheads, corrected for end effects. However, another component to the Gurney equation needs to be added to account for the axial velocity component. A semi-empirical model was proposed in [8], based on similar principles used in the derivation of the Hennequin model, for the prediction of the velocities of fragments, or segments of a disc, which are projected off the opposite end of an end initiated cylindrical charge. Through the introduction of a gas-relaxation term, F_y, the open-faced sandwich Gurney formula can then be written as:

$$v_s = \sqrt{2E}\left[\frac{1}{3}\left(\frac{4M}{CF_y}+1\right)\left(\frac{M}{CF_y}+1\right)\right]^{-1/2}, \tag{10}$$

where

$$F_y \equiv A\left(\frac{M}{C},\frac{L}{D}\right)B(y),$$

with

$$A\left(\frac{M}{C},\frac{L}{D}\right) \equiv 1-\exp\left\{-k\left(\frac{C}{M}\right)^l\left(\frac{D}{L}\right)^m\right\}, \qquad B(y)\equiv\left\{1-\frac{2y}{D}\right\}^n,$$

and *k, l, m* and *n* positive empirical constants.

Eq. (1) and eq. (10) can be considered to be two limiting conditions, i.e. for fragments on the circumference of a cylindrical charge and for fragments on a cylindrical charge in an open-face sandwich configuration, respectively. It is therefore proposed that a single expression for the prediction of the fragment velocities is obtained by combining eqs. (1) and (10) through asymptote matching, namely

$$v = \left(v_c^n + v_s^n\right)^{1/n}. \tag{14}$$

In Fig. 3 the velocities according eq (14) are shown for various values of the shifting parameter *n*. Also shown in the figure are the two asymptotes, i.e. the velocities in the radial and axial directions respectively.

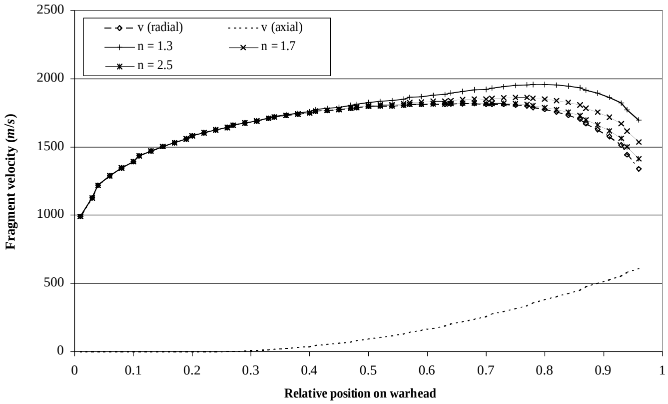

Figure 3: Velocities according Method 2: Combined open-face sandwich and cylindrical configurations.

CONCLUDING REMARKS

In Fig. 4 the various models are compared to experimental data. It is clear from the results, that even for very high values of the shifting parameter (n = 8), Method 1, i.e. with the combined *C/M* representation, gives unrealistic results. The reason for this is that in the convex section of the warhead the axial *C/M* starts dominating, and using this relatively large value of *C/M* in the cylindrical Gurney equation, eq. (1), predicts too large velocities. The velocity predictions according Method 2, i.e. the combination of the open-face sandwich and cylindrical configurations by means of asymptote matching, matched the test data relatively good. This can to a large extent be expected because the shifting parameter n was used to obtain a relatively good transition between the two limiting conditions. Additional test data and maybe data from finite-difference modelling are, however, needed to evaluate the general applicability of eq. (14).

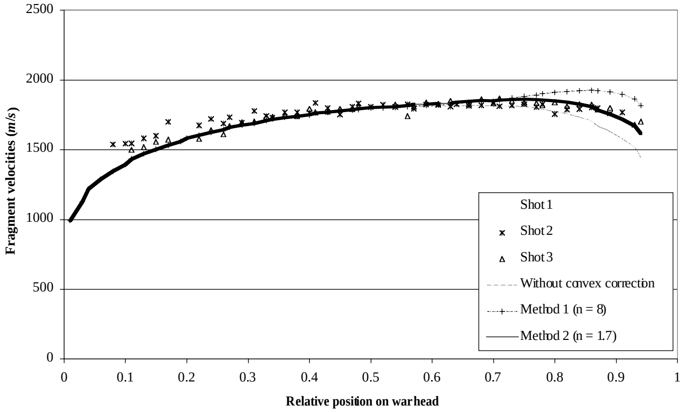

Figure 4: Comparison with experimental results.

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