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SOME IMPROVEMENTS INTO ANALYTICAL MODELS OF SHAPED CHARGE JET FORMATION

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Some improvements have been proposed into analytical models of shaped charge jet formation. The maximum velocity of the liner is estimated by the use of the Gurney type formula for the open cylindrical sandwich with modifications taking into account the influence of the shell. A new expression for the velocity history is proposed. The equations of the trajectories of liner elements are numerically integrated. Calculated profiles of collapsing liners agree well with results of experiments and hydrocode calculations. A new purely geometric criterion of the choice of the moment, at which a given liner element enters the collision zone has been derived. A gradual formation of the leading particle by interactions of adjacent jet particles is considered. A reasonable agreement between velocity distributions in the jet calculated by the analytical model and a hydrocode has been obtained.

INTRODUCTION

Analytical models of shaped charge jet formation are based on the hydrodynamic theory, assuming incompressible liner material, stationary process and plain symmetry. None of the assumptions of the hydrodynamic theory are valid in reference to metal, conical liners and nonstationary process of jet formation. However, the analytical models are willingly used for the sake of their low computational cost in comparison with hydrocodes [1]-[4]. In this work results of an analysis of various elements of analytical models proposed in the literature are reported. The analysis was performed by making comparison of the shapes of collapsing liner, jet and slug calculated by the analytical model, determined experimentally and calculated by a hydrocode [5]. Some improvements into existing analytical models are proposed in order to achieve better agreement with the reference data. The improvements concern the formula for the maximum liner element velocity, the expression describing time dependence of the velocity, the criterion of choosing the moment, when a given liner element enters the collision zone, and treating of the inverse velocity gradient region.

DESCRIPTION OF THE MODEL

All the analytical models make use of the basic formulae of the hydrodynamic theory that combine the velocity of a liner element v, the dynamic deflection angle β and the mass of liner element m with the velocities of the jet v_j and the slug v_s, as well as the jet element mass m_j and the slug element mass m_s. In the considered model the following form of the formulae is used

$$v_j = v_x - v_r \operatorname{ctg}(\tfrac{1}{2}\beta)\,,\; v_s = v_x + v_r \operatorname{tg}(\tfrac{1}{2}\beta) \tag{1}$$

$$m_j = m \sin^2(\tfrac{1}{2}\beta)\,,\; m_s = m \cos^2(\tfrac{1}{2}\beta) \tag{2}$$

where v_x , v_r (<0) mean the axial and the radial components of the velocity of a liner element at the moment of entering the collision zone.

The following formula is derived in [6] for the estimation of the maximum liner velocity

$$v_0 = (\mu+1)^{-1}\left[\sqrt{\frac{v_G^2(\mu+1)-(A/C_0)^2}{\frac{3}{2}N(\mu+1)-1}} - \frac{A}{C_0}\right], \quad N = \frac{(b+1)(3b+1)}{(2b+1)^2}, \quad b = \frac{r_s}{r_i} \tag{3}$$

$$\mu = m/C\,, \quad A = 0{,}933\, p_{CJ}\, \tau\, (r_s - r_i)^2 / r_i\,, \quad C_0 = C/\Delta l \tag{4}$$

$$\tau = (0{,}912 \cdot m_0 v_0 + 20{,}7)/\, p_{CJ}\,, \quad m_0 = \frac{m}{\pi \Delta S}$$
$$[\tau] = \mu\text{s}, \; [p_{CJ}] = \text{GPa}, \; [m_0] = \text{kg/m}^2, \; [v_0] = \text{km/s}, \; [r_s, r_i] = \text{mm} \tag{5}$$

where C means explosive mass assigned to a given liner element, r_s, r_i – external and internal radius of the explosive charge, Δl, ΔS – length and surface area of a given liner element, p_{CJ} – detonation pressure. As the quantity A depends on v_0 the above relations can be transformed into a second order algebraic equation and elementary solved in reference to v_0.

The above formula refers to the shaped charge with no shell. In order to take into account the influence of the shell, the following solution was tried. Fig. 1 shows calculated velocity values for a test charge described in [7]. Plot 1 corresponds to the formula (3), plots 2 and 3 to the formulae for the open and the closed plane sandwiches

$$v_0 = v_G[3/(1+5\mu+4\mu^2)]^{1/2} \tag{6}$$

$$v_0 = v_G\{3/[1+3\mu-\omega+(1+3\theta\mu)\omega^2]\}^{1/2}\,, \quad \theta = M/m\,, \quad \omega = (2\mu+1)/(2\theta\mu+1) \tag{7}$$

where M means the shell mass assigned to a given liner element.

It can be seen that the influence of the symmetry is essential only at the liner apex. On the other hand the influence of the shell near the apex is relatively weak (assumed steel shell with the thickness equal to 10% of CD can be considered as very heavy). That is why the maximum velocity can be calculated by the formula (7) and then the influence of the symmetry is considered by subtracting the difference between v_0 values calculated by the formulae (6) and (3)

$$v_0 = v_0(7) - [v_0(6) - v_0(3)] \quad (8)$$

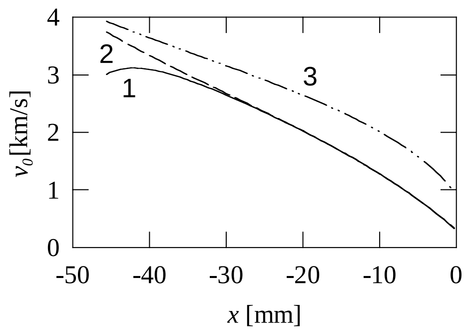

Figure 1: Plots of the maximum liner velocity versus the initial axial position of liner elements ($x = 0$ – liner base), 1 – formula (3), 2 – (6), 3 –(7).

The following expression describing time dependence of the liner velocity was proposed in [6] and is widely used in analytical models

$$v = v_0\{1 - \exp[-(t - T)/\tau]\} \quad (9)$$

where T means the time corresponding to the moment the detonation wave reaches a given liner element, τ is calculated by the use of the formula (5).

The equations of trajectories of liner elements are integrated numerically. Similar approach was used in [4], [8] and [9]. The liner is divided into elements. Locations of the mass centres x_{ci}, r_{ci}, inner and outer liner surfaces x_{wi}, r_{wi}, x_i, r_i, liner element mass m_i, assigned masses of explosive C_i and the shell M_i are calculated – Fig. 2. Then maximum velocities v_{0i}, starting times T_i, time constants τ_i are determined. Initial values of time dependent locations $\{z_{ci}, y_{ci}, z_{wi}, y_{wi}, z_i, y_i\} = \{x_{ci}, r_{ci}, x_{wi}, r_{wi}, x_i, r_i\}$ and velocity components $\{v_{xi}, v_{ri}\} = \{0,0\}$ are put.

The velocity components are determined by making use of the fact that the acceleration vector is normal to the liner surface. Thus, knowing the velocity components at time t we can calculate their approximate values at $t + \Delta t$ by the formulae

$$v_{xi}(t+\Delta t) = v_{xi}(t) + \Delta v \cos\beta(t), \quad \Delta v = v_i(t+\Delta t) - v_i(t) \quad (10)$$

$$v_{ri}(t+\Delta t) = v_{ri}(t) - \Delta v \sin\beta(t) \quad (11)$$

$$\beta(t) = \operatorname{arctg}\{[y_{ci+1}(t) - y_{ci-1}(t)]/[z_{ci+1}(t) - z_{ci-1}(t)]\} \quad (12)$$

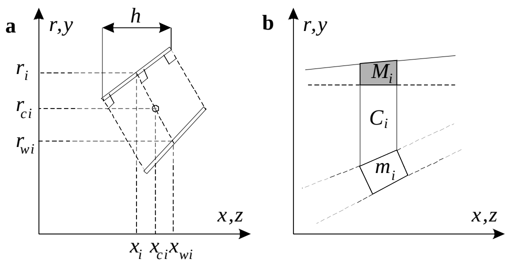

Figure 2: Determination of the locations of the mass centre and the inner and outer surfaces of a liner element (a); assignment of the explosive and shell mass to a liner element (b)

New locations of the mass centres are calculated by the numerical integration

$$z_{ci}(t+\Delta t) = z_{ci}(t) + [v_{xi}(t) + v_{xi}(t+\Delta t)]\Delta t/2 \tag{13}$$

$$y_{ci}(t+\Delta t) = y_{ci}(t) + [v_{ri}(t) + v_{ri}(t+\Delta t)]\Delta t/2 \tag{14}$$

Assuming incompressible material of the liner we can calculate from geometrical relations new locations of the outer and inner liner surfaces z_{wi}, y_{wi}, z_i, y_i.

Choosing the final moment of the liner motion is an essential problem. In [8] and [9] the moment is chosen, when the outer radius of a liner element becomes equal to the estimated slug radius. This approach is chosen as a starting point in the present analysis. In each time step the radius of the slug for the utmost left liner element is estimated by the formula

$$r_{zi} = \cos(\tfrac{1}{2}\beta)q\,, \quad q = \sqrt{\frac{2m_i}{\rho_J \pi \sqrt{(z_{ci+1}-z_{ci-1})^2 + (y_{ci+1}-y_{ci-1})^2}}} \tag{15}$$

where ρ_J means the density of the jet material (in the case of porous liners it is put equal to the density of the monolithic material). When $r_{zi} < y_i$, the utmost left liner element is divided into jet and slug elements. The velocities of the jet and slug are calculated from (1), while the radius of the jet is calculated from

$$r_{ji} = \sin(\tfrac{1}{2}\beta)q \tag{16}$$

Since that moment the motion of the liner and the slug elements is tracked. The initial axial coordinates of their tips and tails are calculated from the formulae

$$z_{zpi} = z_i - \tfrac{1}{2}\Delta l\,, \quad z_{zki} = z_i + \tfrac{1}{2}\Delta l\,, \quad z_{jpi} = z_{wi} + \tfrac{1}{2}\Delta l\,, \quad z_{jki} = z_{wi} - \tfrac{1}{2}\Delta l \tag{17}$$

$$\Delta l = \tfrac{1}{2}\sqrt{(z_{ci+1}-z_{ci-1})^2 + (y_{ci+1}-y_{ci-1})^2} \tag{18}$$

In each time step new positions of jet element tip z_{jpi}, jet element tail z_{jki}, slug element tip z_{zpi} and slug element tail z_{zki} are calculated, assuming that the elements move with a constant velocity. Depending on the relation between velocities of adjacent elements we can have to do with two effects – Fig. 3. The elements can overlap each other or a gap between them can form. In the first case locations of the elements tips and tails, their radii and velocities are changed. We assume that their velocities become equal (inelastic collision). From the conservation of momentum law we obtain

$$v_{ji} = v_{ji+1} = \frac{(z_{jpi} - z_{jki})r_{ji}^2 v_{ji} + (z_{jpi+1} - z_{jki+1})r_{ji+1}^2 v_{ij+1}}{(z_{jpi} - z_{jki})r_{ji}^2 + (z_{jpi+1} - z_{jki+1})r_{ji+1}^2} \quad (18)$$

$$v_{zi} = v_{zi+1} = \frac{(z_{zki} - z_{zpi})r_{zi}^2 v_{zi} + (z_{zki+1} - z_{zpi+1})r_{zi+1}^2 v_{zj+1}}{(z_{zki} - z_{zpi})r_{ji}^2 + (z_{zki+1} - z_{zpi+1})r_{zi+1}^2} \quad (19)$$

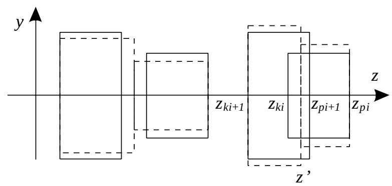

Figure 3: Possible interactions of jet or slug elements

New radii of elements are calculated assuming that after the collision both elements have equal length, values of z_{pi} i z_{ki+1} do not change and new mass of a given element is equal to the total mass between cross sections *z'* and z_{pi} (element *i*) and z_{ki+1} and *z'* (element *i+1*). In the case of a gap, lengths and radii are only changed. The same elongation and constant volumes of each element are assumed.

ANALYSIS OF THE MODEL AND ITS MODIFICATIONS

Fig. 4 shows shapes of a collapsing liner and a slug claculated for a test charge by the use of the described model. They are compared with the results of hydrocode calculations and experiment from [7]. It can be seen that the analytical model forecasts too slow motion of the liner. Obtained results suggest that the expression (9) for the velocity history should be modified. Taking into account that the formulae for the plane symmetry proved to be valid for the most part of the liner, a simple model was considered of two plates launched in the opposite directions by an ideal gas charge with the polytropic exponent equal 3. The uniform pressure field between plates was assumed. The following formula describing time changes of the liner velocity was derived

$$v = v_0\sqrt{1 - \frac{1}{1 + [(t-T)/\tau]^2}}\,, \quad \tau = \frac{m_0 v_0}{p_{CJ}} \quad (20)$$

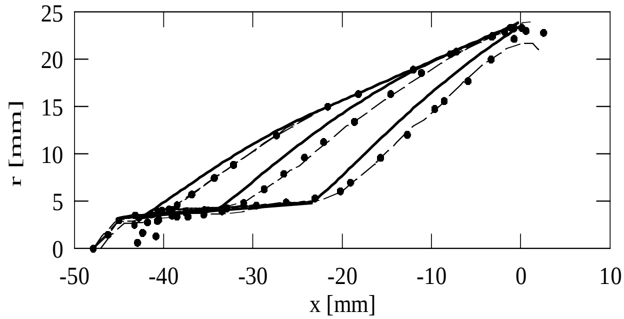

Figure 4: Collapsing liner and slug shapes for t = 8.7, 10.7 and 13.7 µs; solid line – analytical model, dashed line – hydrocode [7], points – experiment [7]; $v(t)$ – (9)

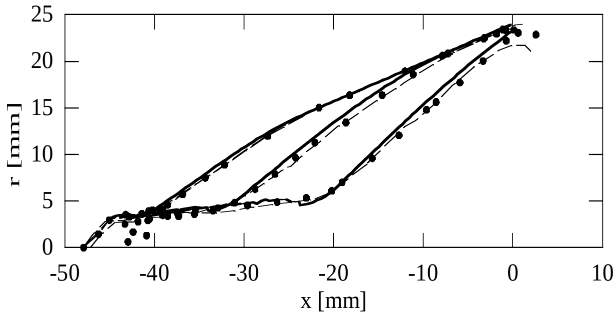

Figure 5: Collapsing liner and slug shapes for t = 8.7, 10.7 and 13.7 µs; solid line – analytical model, dashed line – hydrocode [7], points – experiment [7]; v(t) – (20)

Eq. (20) gives somewhat faster velocity increment than eq. (9). Moreover, the time constant τ calculated from (20) is lower than calculated from (5). That is why, a faster motion of the liner is forecasted by the use of (20). As a result, shapes of the collapsing liner better agree with the hydrocode results and experimental records – Fig. 5.

The analysis of the results of calculations revealed that the location of the inner surface of the liner at the moment of jet formation was closer to the axis than estimated liner radius. Therefore, the condition for the choice of the final moment of the liner motion should be modified. A new condition was derived basing on the following consideration. The parts of a liner element forming the jet and the slug move inside the collision zone along some curved trajectories – Fig. 6. The force acting on a given element in the collision zone is a centripetal force

$$F = mv^2 / R \tag{21}$$

Since $F=pS$, $m=\rho Sd$, where S –surface area of an element, p – pressure, ρ – density, d – element thickness, we have

$$R = \rho\, dv^2 / R \tag{22}$$

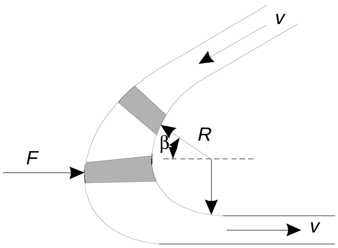

Figure 6: Motion of elements forming the jet in the collision zone

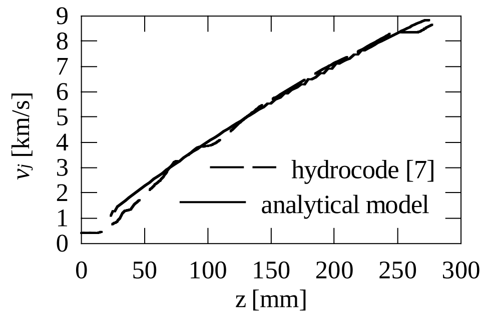

Figure 7: Calculated velocity distributions in the jet

Assuming $p \sim \rho v^2$, we come to a purely geometric condition $R \sim d$. Making use of (22) and taking typical values p = 50 GPa, v = 4000 m/s, ρ = 8900 kg/m^3, we obtain $k_R = R \,/\, d\, p \approx 2{,}8$. The value $k_R = 3$ was assumed. The thickness of an element forming the jet when entering the collision zone was accepted for the value of d. The condition for the determination of the final moment of the liner element motion was obtained

$$\begin{aligned} y_w &\le r_j + k_R(1+\cos\beta)\left(\sqrt{y_{wi}^2 + q_R\cos\beta} - y_{wi}\right)/\cos\beta \\ q_R &= (y_i + y_{wi})\sqrt{(y_i - y_{wi})^2 + (z_i - z_{wi})^2}\,\sin^2(\beta/2) \end{aligned} \tag{23}$$

Since that moment the motion of liner material forming the jet and the slug is modelled as the circular uniform motion. The element forming the jet makes 180°-β angle rotation, while the element forming the slug makes β angle rotation. After rotation, they move as the jet and the slug elements.

SUMMARY

The performed analysis showed that some concepts of the analytical models proposed in the literature should be modified in order to attain better agreement with the results of hydrocode calculations and experiments. A method of estimating the maximum liner velocity was proposed on the basis of existing formulae for the plain and cylindrical open sandwiches. A new formula describing time dependence of the liner velocity was proposed. The new formula gives a reasonable agreement between calculated and recorded shapes of the collapsing liner. A new, purely geometric condition for the determination of the final moment of the liner motion was derived. It provides a realistic assessment of the moment, at which a jet element emerges from the collision zone. As a result, a reasonable agreement between velocity distributions in the jet calculated by the analytical model and hydrocode has been obtained – Fig. 7. A broader analysis of the modified model will be given in a separate publication.

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